Progression in Mental Mathematics

Primary Maths

Herts for Learning

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Contents

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HfL Progression in Mental Mathematics Guidance

Rationale

This document was written in response to the heightened demands of the National Curriculum (2014). It aims to support teachers and leaders with a map of progression in mental fluency that is underpinned by research. The programme of study includes references to mental calculation but lacks the detail needed to provide a coherent pathway. It is up to schools to decide upon what this should look like. This guidance document provides the necessary detail.

With the expectation that pupils will move at ‘broadly the same pace’ through the curriculum, schools have been seeking a pathway from EYFS to Year 6 that will support the development of all pupils’ mental skills, including those who might previously have struggled to develop the core knowledge and understanding necessary. This document reflects the findings of our research projects and ‘tried and tested’ approaches with schools, focusing on practice that is most effective at increasing mental fluency for more of our pupils.

Approaches

Working in collaboration with teachers, Herts for Learning advisers have found that the following approaches have had significant impact upon pupils' ability to develop number sense and multi-strategy approaches to mental calculation.

- **A risk-free environment where learning is valued over performance**

Where the environment praises speed and 'first to get the answer right', it emphasises a competitive view of mathematics. Unfortunately, this has the effect of 'hiding' how fluency is developing in other pupils and implies that mental calculation is a performance. This can adversely affect pupils' desire to engage. Instead, we promote a range of approaches that are more effective in engaging pupils to discuss and reason about their strategies. A risk-free classroom has an ethos that is underpinned by the following attributes:

1. *Everyone* has something to contribute and we *all* value those contributions
2. An appreciation that we each see things differently – there may be one answer, but there are a myriad of available journeys
3. This is not about guessing what is in the teacher's head
4. There is an expectation that we have to try to communicate our ideas so that everyone else can understand them and that we are expected to try and understand the thinking of others
5. There is an expectation that we have to listen to what others say and then try to build on it - agreeing and disagreeing by offering proof.

All of the following approaches can be utilised in shared, whole-class learning discussions. It is not an exhaustive list but provides a flavour of available possibilities.

1. Give the calculation *and* the answer – shared class discussion: 'How could you do this?'
2. 'One finger, one way' – show me your thumb when you have found one way to find the answer, keep thinking and show me another finger when you have found another way…
3. 'Show me, show me' – show me your thinking in as many ways as possible.
4. 'Can you use the ________ *(named)* strategy to solve this?'
5. 'Cluster of facts' – pupils identify facts that would be helpful to solve a calculation and / or explain why given facts might be useful to solve the problem.
6. 'Shortcuts' – "I could take a shortcut in this strategy if I …"
7. 'Seek and destroy' – identify correct and incorrect answers from a range and explain why.

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- **CPA (concrete-pictorial-abstract)**

To understand the numbers they are working securely with and develop number concepts alongside the procedures, the CPA approach allows pupils to demonstrate and explore learning across a range of representations.

For example, when very young pupils learn about ‘3 + 2’ they need to learn that the symbols stand for the operation of addition i.e. adding 2 to 3. They also need to understand the concept of a sum. In the CPA approach, pupils would explore the calculation using **concrete apparatus** to identify the ‘2’, the ‘3’ and the sum 5 *as well as* **pictorial representations** of the same calculation and the **abstract notation** (including language) to better understand both the procedure of adding 2 to 3 *and* the idea of sum.

3 + 2 = 5

“3 cubes and 2 cubes equal 5 cubes”

“3 cubes and 2 more cubes is 5 cubes”

“five is two more than three”

“three girls and two boys is five children altogether”

“3 girls and 2 more girls makes 5 girls”

“What other facts could we use to make 5?”

Without exploration through a range of representations, we cannot expect pupils to develop a full understanding of the underpinning concepts, facts and skills that are integral to developing good mental fluency. Schools should decide on core representations. They also need to ensure that variations of these are also included so that once pupils are able to they can be supported to assimilate learning to new representations.

- **Practice**

Practice is a key approach to developing the automaticity needed to reduce cognitive load. Pupils who have facts and skills at their fingertips are more likely to attend to the particulars of new learning than those that do not. These pupils have to work harder and are over-burdened. At Herts for Learning, we think of practice not as meaningless repetition of facts in which pupils chant without thought or as a series of isolated facts learnt at home then tested in school, but as a chance to rehearse them within exercises that develop better thinking. Practice is an opportunity to keep facts and skills 'simmering' and a further chance to vary the ways that they are presented. Schools should be mindful of the *quality* of practice rather than the *quantity*. Similarly, they are advised to focus upon the facts and skills that will make the greatest difference to mental fluency at each phase.

- **Facts to be practised**

At the end of each of phase, further guidance identifies which facts will support fluency. For example:

Year Three Recall

- Sums and differences between pairs of numbers which are multiples of 10 and 100
- Doubles and halves of multiples of 10 or 100
- Complements to 100
- Complements to 60 (time)
- Complements of tenths that make 1
- Complements of fractions with the same denominator that make 1 e.g. 3/7 + 4/7 = 1
- x 3, x 4, x 8 multiplication facts including division facts
- Number of seconds in a minute
- Number of days in a month and in a year including a leap year

- **Skills to be practised**

HfL advisers have identified a selection of key skills that, when practised, lead to increased mental fluency. Alongside increasing fact acquisition, they allow pupils to develop greater access to choices of strategy.

These are denoted in the progression document by a blue lozenge **Core skill: REGROUPING** and are defined below.

Subitising	the ability to see number as pattern, such as dice patterns. This supports pupils to see numbers within numbers and better regrouping (partitioning).
Regrouping (partitioning)	the ability to break numbers up and recombine them flexibly
Counting on and counting back	in a variety of interval steps
Reordering	knowing when and how to reorder to make calculations easier
Finding complements	links to reordering, identifying useful complements pairs or trios of 1, 10, 60 etc.
Applying the inverse	use of fact family knowledge to 'undo'
Rounding	to a range of benchmark numbers
Estimation	both linear estimation on number lines and scales, and of quantities and calculations to support an increasing sense of what is reasonable
Compensation	to use rounding to add or subtract too much or too little and adjust accordingly
Rebalancing	to adjust the parts of addition and subtraction facts to make a calculation easier
x ÷ by powers of 10	
Doubling and halving	
Rearranging	to adjust the groups in multiplication and division to make a calculation easier

- **Core concepts**

Secure mental fluency is dependent upon a range of underpinning concepts that develop over the primary phase. These are identified in the progression document by a purple box.

Core concept: UNITISING

We recommend that schools monitor how these concepts are evident through the school and how they build progressively through each phase.

Counting concepts

- ***one-one principle*** – each object counted once and given one counting tag
- ***stable-order principle*** – we use the words in the same order
- ***cardinal principle*** – the last number counted is the number of the set
- ***abstraction principle*** - counting can apply to objects which are not tangible e.g. number of claps
- ***order-irrelevance principle*** - it doesn't matter which order we count objects in

Conservation

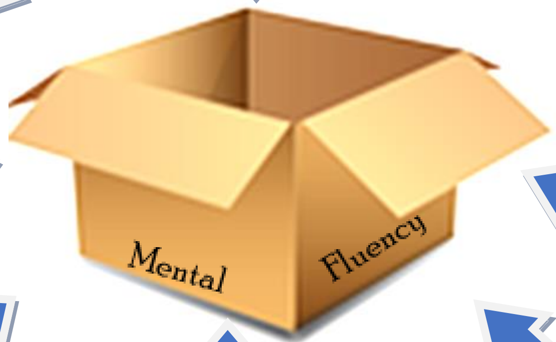
Unless we increase or decrease the set, it will stay the same. The quantity is conserved. For example, we can muddle up 6 counters after counting but there will still be 6 counters.
This is crucial for the concepts of sum and commutativity.

Sum

The total of quantities combined. The concept of equal sum is linked to conservation. We can rebalance the quantities combined and the sum will stay equal.
For example, 1 + 3 = 2 + 2

Equals

Where two expressions have the same value as each other. Knowing that two quantities that look different may be equivalent in value.



Magnitude

Understanding the size of numbers, including their proportional relation to other numbers.
A pre-requisite skill for comparison.

Unit

The idea that individual items can be grouped together to make a new unit. For example, two socks make a pair; ten ones become one ten.
This concept is crucial for place value.

Place Value Concept

Individual items can be grouped and thought of as a single unit. 10 ones = 1 ten (see concept unit)
Units of ones, tens and hundreds can be taken apart and regrouped in different ways.
The position of digits in a number denotes value.
Scaling by powers of 10.

Difference

The answer to a subtraction calculation. Comparison between two values or quantities. Also linked to the concept of magnitude and distance between numbers.

Scaling

A multiplicative concept in which a unit or quantity is compared to a proportionally greater or smaller amount. For example, twice as much cream, 4 pairs of socks, half of 8.

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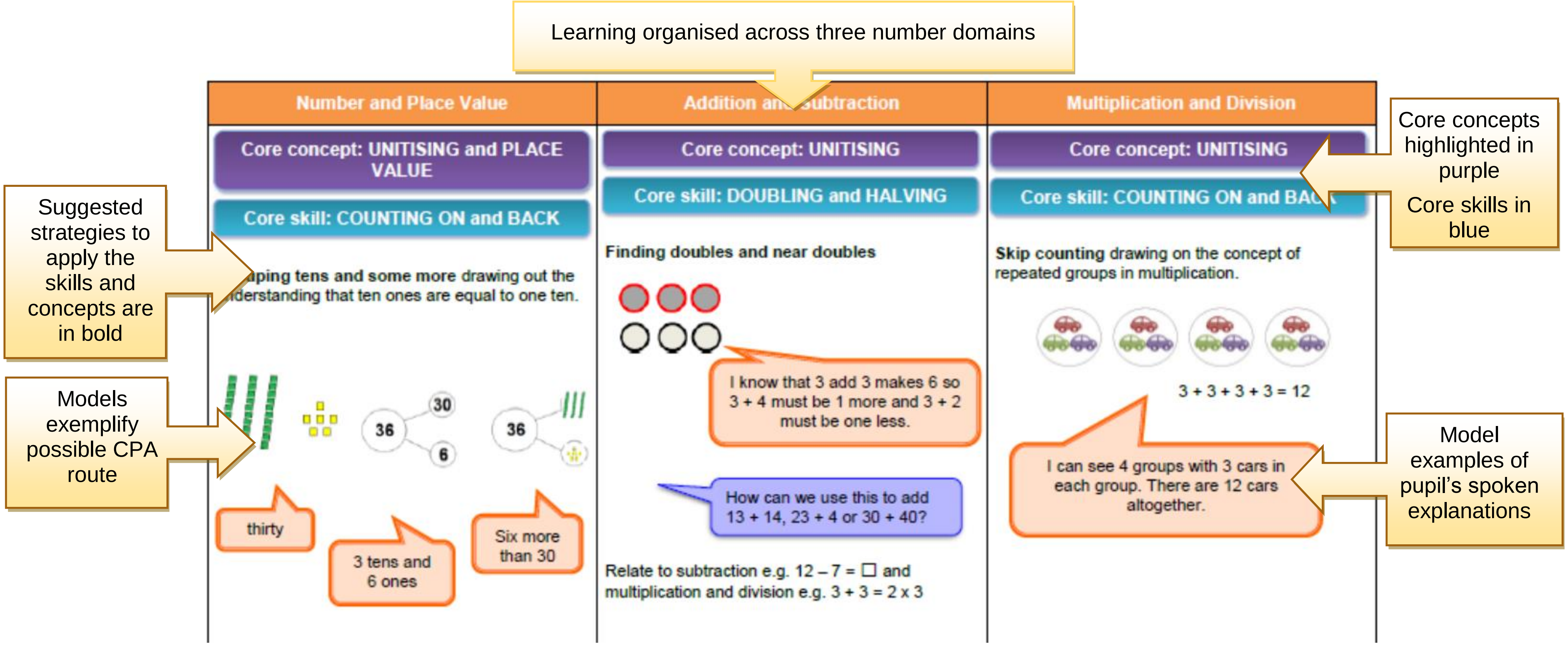
Using the progression document

The progression is structured into phases. In Years 1 to 4, this is organised into individual year groups. At the beginning, there is a section entitled ‘Pre-operational Learning’. This helps ensure that the foundations are secure by the end of EYFS and in the first few weeks in Year 1 before mental fluency within numbers to 10 begins. This also supports the early identification of gaps and barriers.

In each year group / phase, the progression is organised into the National Curriculum Programs of Study domains: number and place value; addition and subtraction; multiplication and division including fractions. Within these domains, key **concepts** (ideas), **skills** (which can be utilised) and **strategies** (methods) are exemplified within the relevant number ranges.



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At the end of each phase, a selection of possible examples that align with a given strategy or skill are included. For KS1 and UKS2, there are examples taken directly from the relevant end of key stage assessments (2016) and sample papers. When designing opportunities to practise or for strategy discussions, these will support teachers to explore and / or guide pupils towards a particular strategy.

Upper KS2 examples

Place Value

937 + 100   1969 + 100   546 - 40
1.7 + 0.05   40 000 - 500
246 ÷ 1   100 x 217   0.4 ÷ 10
1.68 x 100   100 x 100

Examples from 2016 KS2 and Sample Papers

435 - 30   979 + 100   3.005 + 6.12   2.15 + 0.05
100 x 412   0.9 ÷ 10   1.28 x 100   50,000 – 500
10 x 100

Two decimal numbers add together to equal 1 One of the numbers is 0.007. What is the other number?

Circle two numbers that added together make 0.25
0.05   0.23   0.2   0.5

Circle two numbers that multiply together to equal 1 million
200   2,000   5,000   50,000

Write the number that is 5 less than 10 million

Write the number that is one hundred thousand less than six million

Compensation

56 + 8   72 + 9   56 - 8   72 - 9
371 + 18   255 + 49   304 + 299
673 - 99   854 - 398   3720 - 996
0.71 + 0.09   0.56 + 0.08   0.34 – 0.09
£1.17 + £0.39   £8.89 - £4.99

Examples from 2016 KS2 and Sample Papers

468 - 9   472 - 9   15.98 + 26.314
12 – 6.01   15.4 - 8.88

Rebalancing - Equal sum

56 + 8   72 + 9   371 + 18   255 + 49
304 + 267
£37.67 + £3.85   563 + 397   890,488 + 4,890
229,899 + 31,321

Examples from 2016 KS2 and Sample Papers

89,994 + 7,643   936 + 285   89,994 + 7,643

Rebalancing - Equal difference

Think Partition for x and ÷

32 x 4   29 x 2   122 x 4   4.6 x 2
75 x 3   8.3 x 6   39 x 7
3.3 x 7   5 x 49   4 x 198   96 x 0.3

Examples from 2016 KS2 and Sample Papers

15 x 6.1   24 x 3   1.52 x 6   7,505 ÷ 5
$17 \times 1\frac{1}{2}$

Make links to doubling and halving

50 x 28   86 x 50   500 x 70   18 x 2.5
86 x 2.5   160 x 35   500 x 88   1.5 x 6.6
0.5 x 120   4.5 x 2.2   15% x 346   75% x 220

Examples from 2016 KS2 and Sample Papers

15% x 440 $\frac{2}{5}$ x 140   24 x 3
20% of 1500   95% of 240

Multiplying and dividing fractions

Examples from 2016 KS2 and Sample Papers

Implementing the progression

Before implementing the progression, schools should consider some or all of the following self-evaluation questions. These will support leaders to identify the most important focuses and actions.

Is practice effective in your school? How do you know?

- As a school, are the principles of effective practice design understood?
- Have you identified the key skills and facts in which automaticity for the majority of pupils is the aim?
- Where are the gaps? What are the barriers to pupils developing secure mental fluency?
- To develop effective mental strategies, where does teacher subject knowledge and understanding of approaches need to be strengthened?
- Is there a common language when discussing mental fluency?
- Which strategies did pupils use in the end of key stage assessments? Were these strategies informed by mental fluency?
- What does progression look like now for each of the number domains in mental fluency?

Implementing any new curriculum focus and related approaches should focus upon the impact on pupil outcomes. This includes both quantitative and qualitative measures. Any development of teaching and learning should have this priority at its centre. Actions need to be specific, matched to intended goals and clear to all stakeholders. Implementation should include opportunities for evidence-based reflection points. The emphasis, here, is evaluative and lessons learned should be shared across the community before the next steps are considered.

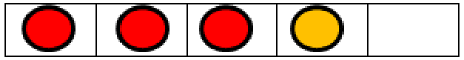
Pre-operational Learning

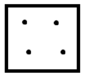
Number and Place Value

Core concept: COUNTING

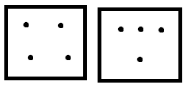
Core skill: SUBITISING

Numbers to 5 recognising dot patterns on dice / dominoes and fives frames.

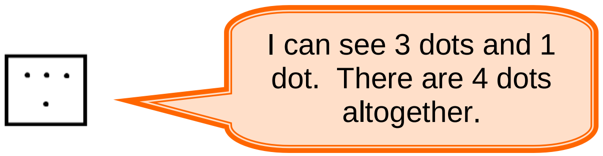




Matching patterns where the number of dots is equal. Progress to patterns where the number of dots are equal but the pattern is different.



Finding dot patterns that are one more or one less than the pattern displayed.

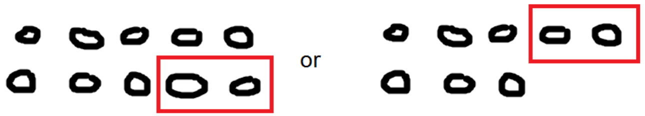


Addition and Subtraction

Core concept: COMPARISON

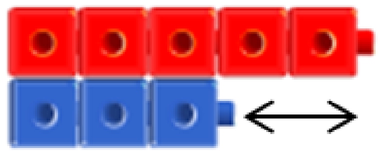
Comparison model within numbers to 10 drawing out language of greater, more, less, fewer than and equal.

Direct comparison



Matching

Discrete cube model

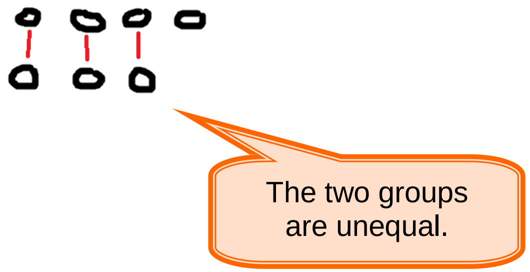


Multiplication and Division

Core concept: COMPARISON

Equal grouping drawing out the concept of equal and unequal amounts.

Identifying when groups of objects are equal or unequal using one to one correspondence.



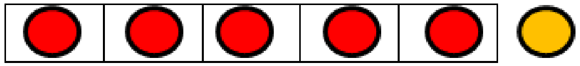
Pre-operational Learning

Number and Place Value

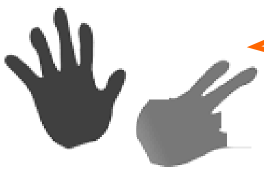
Core concept: COUNTING

Core skill: SUBITISING

Numbers 6 and 7 where 5 is the benchmark i.e. 6 is 1 more than 5 using fives frames and additional counters.



Ensure transference to fingers.

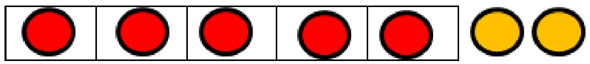


7 fingers can be shown by 5 fingers and 2 more fingers.

Matching patterns where the number of dots is equal. Progress to patterns where the number of dots are equal but the pattern is different.

Finding dot patterns that are one more or one less than the pattern displayed.

Identifying numbers within the whole set of dots (i.e. conceptual subitisation).

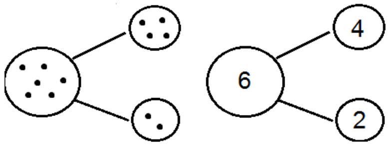


Addition and Subtraction

Core concept: UNITISING

Core skill: REGROUPING

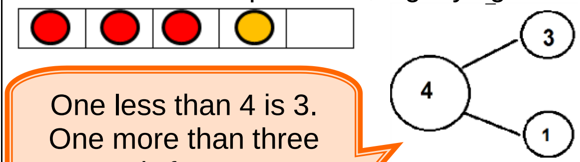
Key model: Part whole model drawing out the understanding that in addition and subtraction there is a relationship between parts and a whole.



Four and two make six.
Two more than four is six.
There is a four and two in six.

One more, one less

Where one of the parts is 1; signifying one more or 1 less.

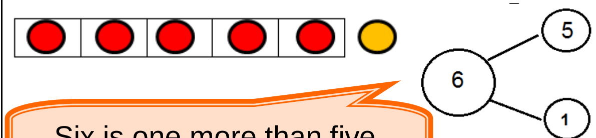


3 + 1 = 4
1 + 3 = 4
4 - 1 = 3
4 - 3 = 1

One less than 4 is 3.
One more than three is four.

Think 5

Where the whole is 6 or 7 and one of the parts is 5 (to secure benchmark from 5).



6 = 5 + 1
6 = 1 + 5
6 - 1 = 5
6 - 5 = 1

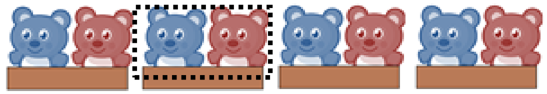
Six is one more than five.
Five is one fewer than six.

Multiplication and Division

Core concept: UNITISING

Patterning drawing out the concept of a repeated unit.

Identifying the part of the pattern that repeats (i.e. identifying the unit being repeated).



Move the pupils through the following steps:

- replicate repeated patterns predict the next part of a repeating pattern
- recognise missing elements of a repeating pattern.

Pre-operational Learning

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Year 1

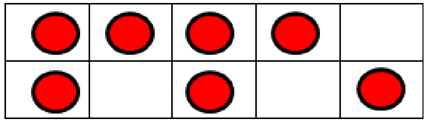
Number and Place Value	Addition and Subtraction	Multiplication and Division

Number and Place Value

Core concept: COUNTING and COMPARISON

Core skill: SUBITISING

Numbers to 10 recognising dot patterns on dice / dominoes and tens frames.



I can see three and three and one makes seven. Four and one and one and one makes seven.

Progression

Matching patterns where number of dots is equal.

Matching patterns where the number of dots is equal but the pattern is arranged differently.

Finding dot patterns that are one more or one less than the pattern displayed.

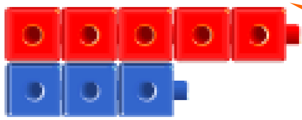
Identifying numbers within the whole set of dots (see example above).

Addition and Subtraction

Core concept: COMPARISON

Core skill: COUNTING ON and BACK

Pupils count on to find the total and difference.

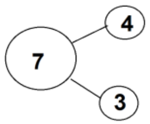


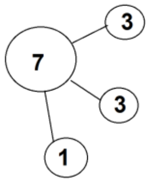
Three and two more equals five. Two fewer than five equals three.

Core concept: CONSERVATION

Core skill: REGROUPING

Part part whole model drawing out an understanding of commutativity.



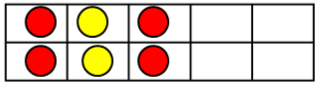


Pupils to extract fact families from the models and explore commutativity.

Multiplication and Division

Core concept: UNITISING

Equal grouping drawing out understanding of repeated addition.



and

2 + 2 + 2 = 6

There are three groups of two teddies. Three groups of two equals six.

Year 1
Numbers to 10

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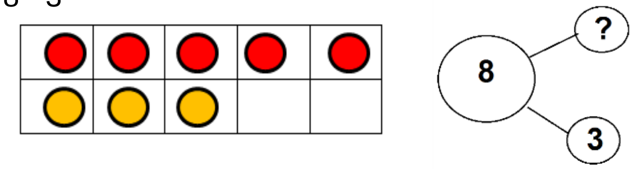
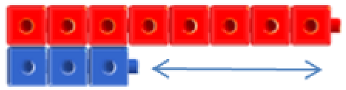
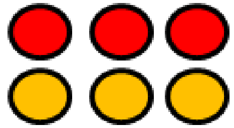
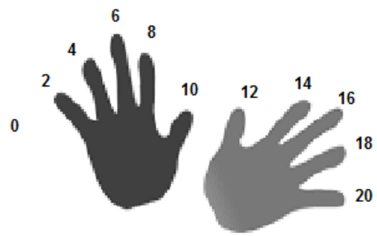
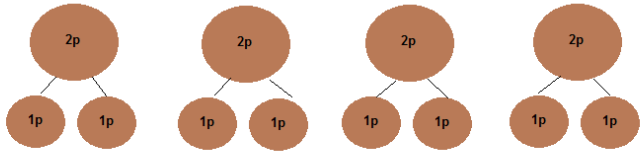
Number and Place Value	Addition and Subtraction	Multiplication and Division
	Core skill: APPLYING THE INVERSE **Think addition** to solve subtraction – leading to greater understanding that if we know one part we can use that to find the unknown part. 8 - 3   I can see that eight can be split into three and five. I know that five and three are eight. I can see that five more than three is eight.	**Core skill: COUNTING ON and BACK** **Skip counting** Counting groups of objects with two hands (drawing out understanding of doubles).  To include opportunities to count in 2s in several ways.  

Year 1
Numbers to 10

Number and Place Value	Addition and Subtraction	Multiplication and Division

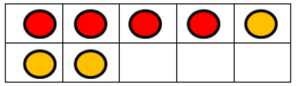
Core concept: CONSERVATION

Core skill: REGROUPING

Think 5 for addition using five as a benchmark number.

4 + 3 =

can become



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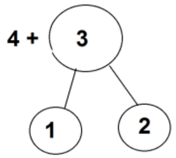

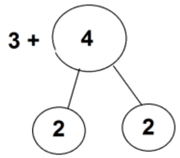

4 + 1 + 2 =
5 + 2 =

3 + 2 + 2 =
5 + 2 =

I know that four and one more is five.
I can see that three can be split into one and two.
Five and two more is seven.

I know that three and two more is five.
I can see that four can be split into two and two.
Five and two more is equal to seven.

Year 1
Numbers to 10

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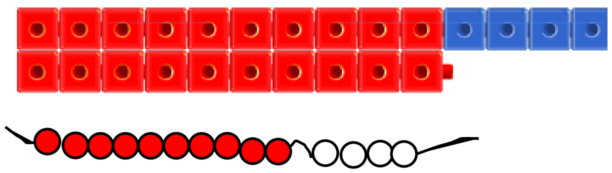
Number and Place Value

Core concept: COMPARISON and PLACE VALUE

Comparing numbers using ten as a benchmark number.



14 - 4 = 10
4 + 10 = 14
10 = 14 - 4
4 = 14 - 10

14 is 4 more than 10.
4 more than 10 is 14.
10 is 4 fewer than 14.
10 fewer than 14 is 4.

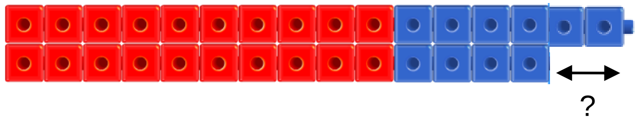
Explore the language of ‘more than’ and ‘less than’ through measures and bar charts.

Addition and Subtraction

Core concept: COMPARISON

Core skill: COUNTING ON and BACK

Comparison model
Pupils to count on to find total and difference.



14 + ☐ = 16

☐ + 14 = 16

16 - 14 = ☐

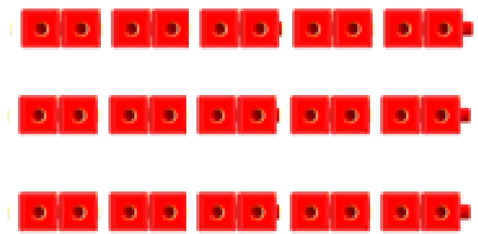
16 - ☐ = 14

14 and 2 more equals 16.
2 fewer than 16 equals 14.

Multiplication and Division

Core concept: UNITISING

Equal grouping drawing out the concept of repeated addition.



10 + 10 + 10
3 groups of 10 equals 30

I can see 10 and 10 and 10 makes 30.
3 equal groups of 10 makes 30.
30 can be split into 3 groups of 10.

Year 1
Numbers to 20

PA Plus

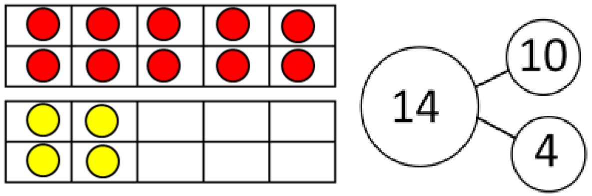
Herts for Learning

Number and Place Value

Core concept: UNITISING and PLACE VALUE

Core skill: REGROUPING

Regrouping numbers into ten and some more drawing out understanding that ten ones are equal to one ten.



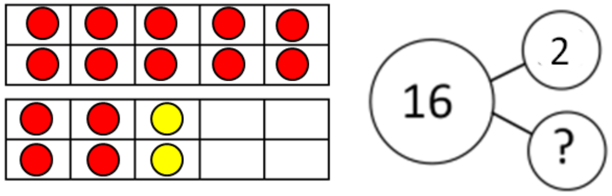
I made 14 into 10 and 4 more.
There is 1 ten and 4 ones in the number 14.
Here is the ten and here are the 4 more.

Addition and Subtraction

Core skill: APPLYING THE INVERSE

Think addition to solve subtraction using the comparison or part whole model to identify the missing part.

16 - 2 = ☐



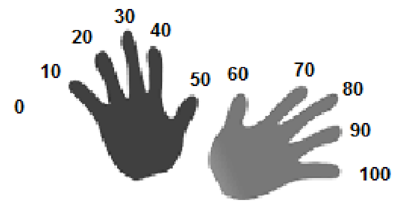
I can see that sixteen can be split into fourteen and two.

I know that fourteen and two more is sixteen.
So 16 subtract two is fourteen.

Multiplication and Division

Core skill: COUNTING ON and BACK

Skip counting
Counting to include opportunities to count in 5s and 10s in several ways including with coins, tallies and pictograms.



Year 1
Numbers to 20

PA Plus

Herts for Learning

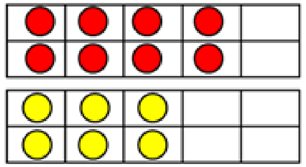
Number and Place Value	Addition and Subtraction	Multiplication and Division

Core concept: CONSERVATION

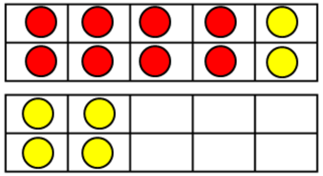
Core skill: REGROUPING

Regrouping numbers to 20 leading to ‘**think 10 for addition’**.
Pupils should experience regrouping either addend.

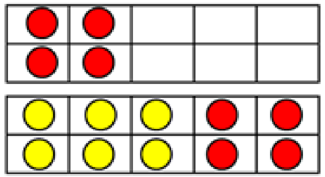
8 + 6

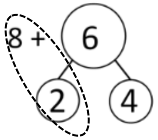


can become

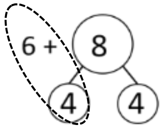


or





8 + 2 + 4 =
10 + 4 =

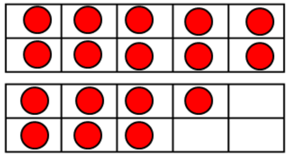
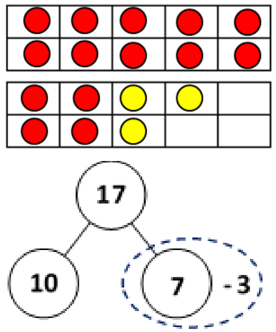
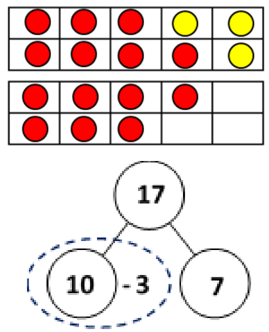


6 + 4 + 4 =
10 + 4 =

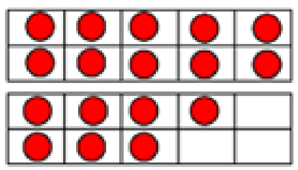
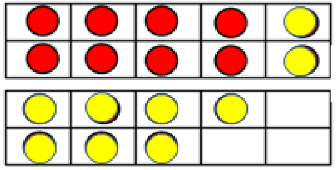
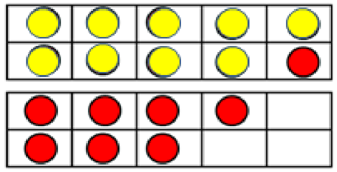
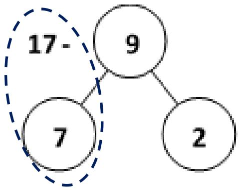
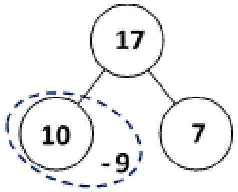
I know that eight and two more is ten. Four and two make six. So 8 + 6 can become 8 + 2 + 4.

I know that six and four more is ten. Four and four make ten. So 6 + 8 can become 6 + 4 + 4.

Year 1
Numbers to 20

Number and Place Value	Addition and Subtraction	Multiplication and Division
	‘Think 10 for subtraction’ using the minuend *or* the subtrahend. First with numbers where no bridging through ten is required. 17 - 3  Regrouping the minuend in two ways and recombining the remaining quantity.  or  17 - 3 = 7 - 3 + 10 = 4 + 10 = 14 17 - 3 = 10 - 3 + 7 = 7 + 7 = 14 I know that 17 can be regrouped into 10 and 7. I can take 3 from either 10 or 7.	

Year 1
Numbers to 20

Number and Place Value	Addition and Subtraction	Multiplication and Division
	Then with numbers which would require bridging through ten. For example, 17 - 9  **Regrouping the subtrahend** or **Regrouping the minuend**  or   17 - 9 = 17 - 7 - 2 = 10 - 2 = 8  17 - 9 = 10 - 9 + 7 = 1 + 7 = 8 Nine can be regrouped into 7 and 2. I can take 7 from 17 to leave 10 and then I can use my number bonds to take away 2 more. Seventeen can be regrouped into 10 and 7. Then I can use my number bonds to take 9 from 10. I'm left with 1. Then I add one to seven.	

Year 1
Numbers to 20

PA Plus

Herts for Learning

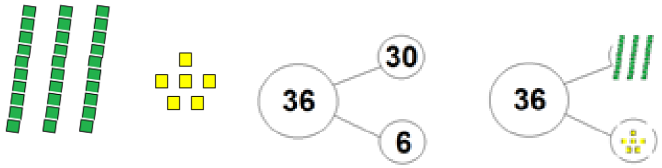
Year 2

Number and Place Value

Core concept: UNITISING and PLACE VALUE

Core skill: COUNTING ON and BACK

Grouping tens and some more drawing out the understanding that ten ones are equal to one ten.



thirty-six

3 tens and 6 ones

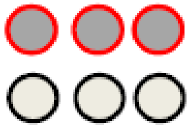
6 more than 30

Addition and Subtraction

Core concept: UNITISING

Core skill: DOUBLING and HALVING

Finding doubles and near doubles



I know that 3 add 3 makes 6.
So 3 + 4 must be 1 more.
3 + 2 must be one less.

How can we use this to add 13 + 14, 23 + 4 or 30 + 40?

Relate to subtraction e.g. 12 - 7 = ☐ and multiplication and division e.g. 3 + 3 = 2 x 3.

Multiplication and Division

Core concept: UNITISING

Core skill: COUNTING ON and BACK

Skip counting drawing on the concept of repeated groups in multiplication.

3 + 3 + 3 + 3 = 12

I can see 4 groups with 3 cars in each group. There are 12 cars altogether.

Year 2
Numbers to 100

PA Plus

Herts for Learning

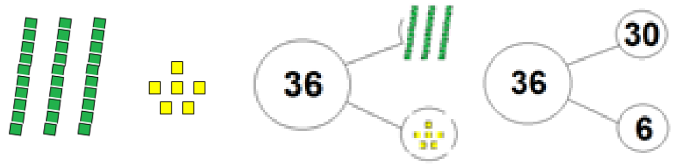
Number and Place Value	Addition and Subtraction	Multiplication and Division

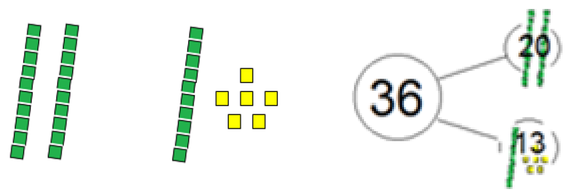
Number and Place Value

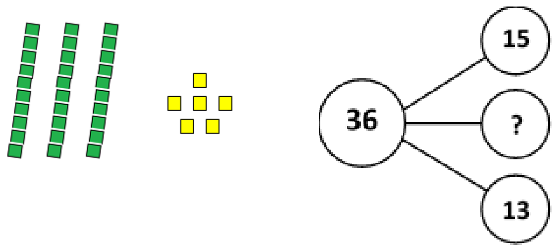
Core concept: CONSERVATION and PLACE VALUE

Core skill: REGROUPING

Regroup 2-digit numbers flexibly and in multiple ways.







Addition and Subtraction

Core concept: CONSERVATION

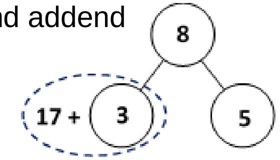
Core skill: REGROUPING

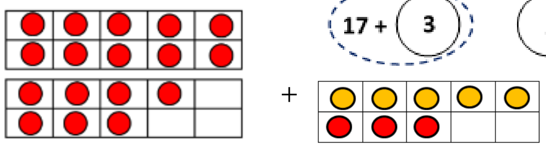
Think 10 for addition (Tens Ones + Ones)

Drawing out the skill of regrouping numbers to allow bridging through tens: Tens Ones + Ones. Exploring that either addend can be regrouped and utilise benchmark numbers.

□ = 17 + 8

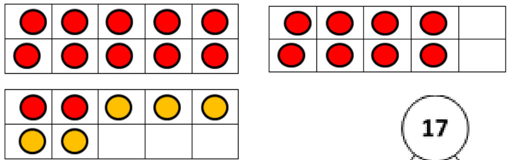
a) Regrouping the second addend

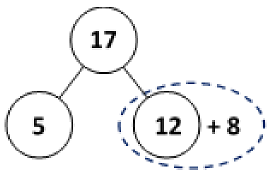




17 + 8 =
17 + 3 + 5 =
20 + 5 = 25

b) Regrouping the first addend





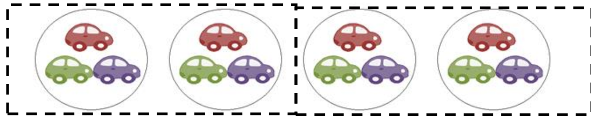
17 + 8 =
12 + 8 + 5 =
20 + 5 = 25

Multiplication and Division

Core concept: UNITISING and SCALING

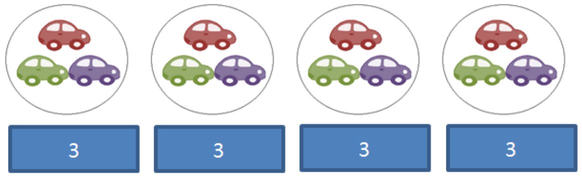
Core skill: DOUBLING and HALVING

Doubles

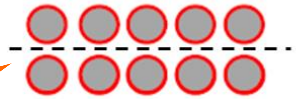


I can also see two groups of six.

Leading to linear and area models.



There are 4 groups of 3 cars. That is 12 cars altogether. I can see 12 cars with 4 groups of 3 cars.



2 x 5 is equal to double 1 x 5.

Year 2
Numbers to 100

PA Plus

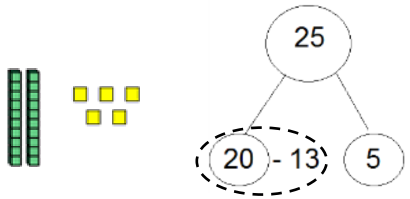
Herts for Learning

<table>
<tr><th>Number and Place Value</th><th>Addition and Subtraction</th><th>Multiplication and Division</th></tr>
<tr><td></td><td>

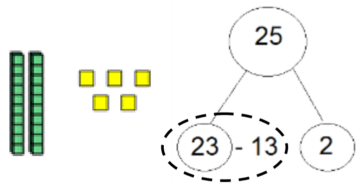
Think 10 for subtraction (Tens Ones - Ones)
Exploring that either the minuend or the subtrahend can be regrouped.

25 - 13 = ☐

Regrouping the minuend (two examples). Taking from a multiple of ten or taking to a multiple of ten.

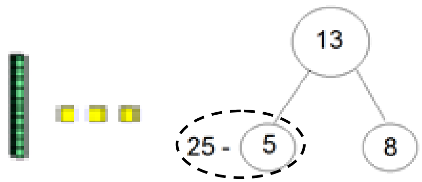


20 - 13 + 5
= 7 + 5
= 12



23 - 13 + 2
= 10 + 2
= 12

Regrouping the subtrahend – normally to a multiple of ten.



25 - 5 - 8
= 20 - 8
= 12

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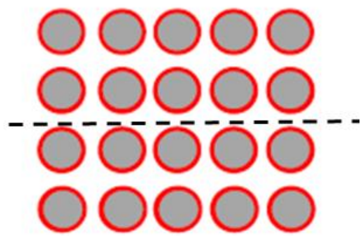
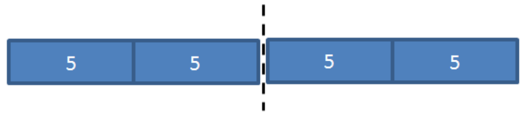




4 x 5 is double 2 x 5.
I can show it as an array and as a linear model.

</td></tr>
</table>

Year 2
Numbers to 100

PA Plus

Herts for Learning

Number and Place Value	Addition and Subtraction	Multiplication and Division

Addition and Subtraction

Core concept: UNITISING

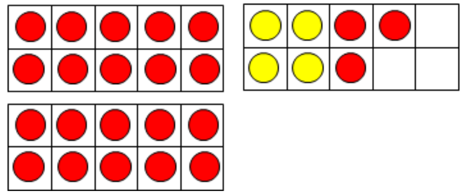
Core skill: FINDING COMPLEMENTS / REORDERING

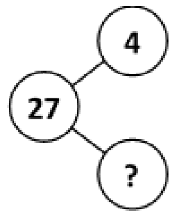
Adding three, one-digit numbers such as 7 + 5 + 6 or 6 + 7 + 4 and drawing out the reasons why pupils may wish to reorder the numbers. Focus upon a range of strategies used.

Core skill: APPLYING THE INVERSE

Think addition to solve subtraction

Tens Ones - Ones = ☐ without regrouping





27 - ☐ = 4

4 + ☐ = 27

I know 4 and 3 makes 7 so 4 and 23 make 27.

Multiplication and Division

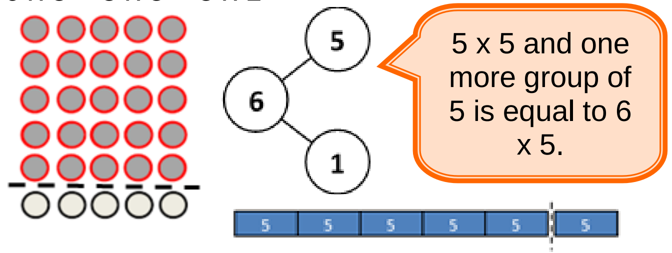
Core concept: CONSERVATION

Core skill: REGROUPING

Applying understanding of benchmark numbers.

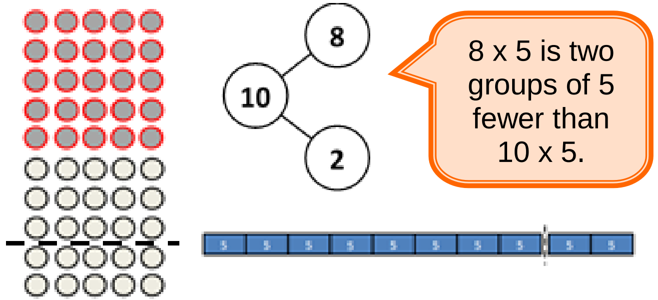
Think 5 for multiplication and division

6 x 5 = 5 x 5 + 5 x 1



Think 10 for multiplication and division

8 x 5 = 10 x 5 - 2 x 5



Year 2
Numbers to 100

Number and Place Value	Addition and Subtraction	Multiplication and Division

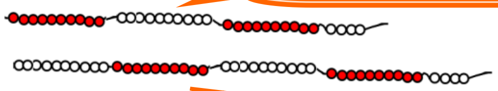
Number and Place Value

Core concept: COUNTING and PLACE VALUE

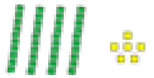
Core skill: COUNTING ON and BACK

Counting on and back in ones and tens from any number allowing children to notice in the pattern what changes as a result (and what doesn't change).

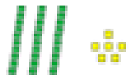
34 3 tens and 4 ones



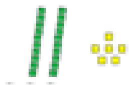
44 4 tens and 4 ones



46 4 tens and 6 ones



36 3 tens and 6 ones



26 2 tens and 6 ones

I can see that the tens are changing but the ones are staying the same.

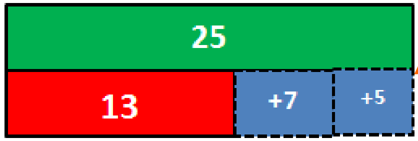
Then counting with coins and on scales from any amount.

Addition and Subtraction

Core concept: COMPARISON

Core skill: COUNTING ON and BACK

Drawing out complements to benchmark numbers.

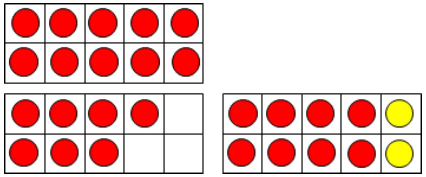


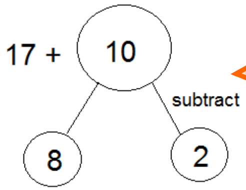
I know there is 7 more to 20 and 5 more to 25.

Core skill: COMPENSATION

Compensation at this stage is a form of **Think 10**, utilising benchmark numbers.

17 + 8 = ☐





Adding 8 is like adding ten and taking 2 away. Subtracting 8 is like subtracting ten and adding 2 back.

Apply this to subtraction.

Multiplication and Division

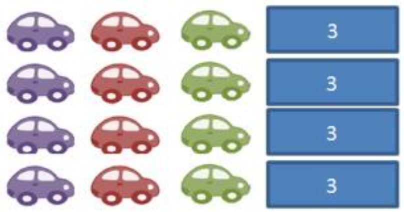
Core skill: APPLYING THE INVERSE

Think multiplication
Introduction of the array and linear model to explore how the relationship of multiplication and division relate.

In multiplication, explore how multiplier, multiplicand and product interrelate.

In division, explore how dividend, divisor and quotient interrelate and link to multiplication.

4 x 3 = 12



Number in each group
3

Number of groups
4

4 x 3 = 12
3 x 4 = 12
12 ÷ 3 = 4
12 ÷ 4 = 3

Year 2
Numbers to 100

PA Plus

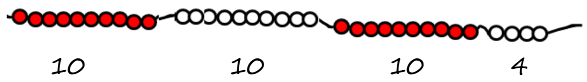
Herts for Learning

Number and Place Value	Addition and Subtraction	Multiplication and Division

Number and Place Value

Core concept: COMPARISON and PLACE VALUE

Comparing numbers using tens as **benchmark numbers.**



34 - 4 = 30
4 + 30 = 34
30 = 34 - 4
4 = 34 - 30

34 is 4 more than 30.
4 more than 30 is 34.
30 is 4 fewer than 34.
30 fewer than 34 is 4.

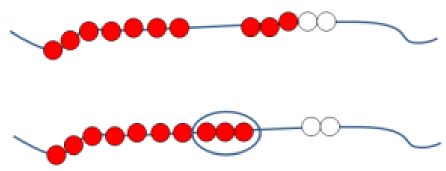
Explore the language of ‘more than’ and ‘less than’ through measures and bar charts.

Addition and Subtraction

Core concept: CONSERVATION and SUM

Core skill: REBALANCING

Equal Sum drawing out understanding that the sum remains equal when we rebalance the addends in an addition calculation.



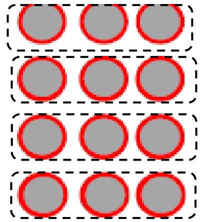
I can prove that 7 + 5 = 10 + 2 using a bead string.

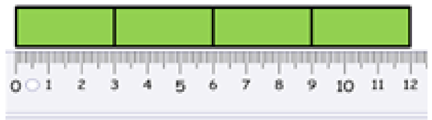
Multiplication and Division

…continued

12 ÷ 3 = 4

I can see that 3 can be taken from 12, four times.





I can see that 12 can be shared into 4 equal groups with 3 in each group.

I know that I can use 4 x 3 = 12 to answer 12 ÷ 3 or 12 ÷ 4.

Year 2
Numbers to 100

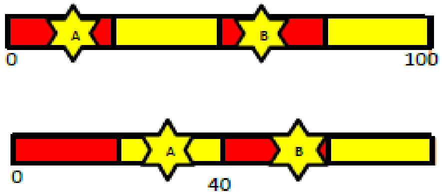
PA Plus

Herts for Learning

Number and Place Value

Core concept: MAGNITUDE

Core skill: ESTIMATION



Drawing out understanding of the distance of numbers to target numbers.

24 is 4 away from 20 and 6 away from 30. 24 is nearer to 20 than to 30.

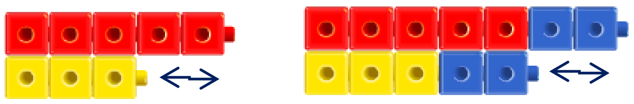
Addition and Subtraction

Core concept: COMPARISON

Core skill: REBALANCING

Equal Difference drawing out understanding that adding or subtracting the same quantity from both the subtrahend and minuend maintains the difference between the numbers.

5 - 3 is equal to 7 - 5

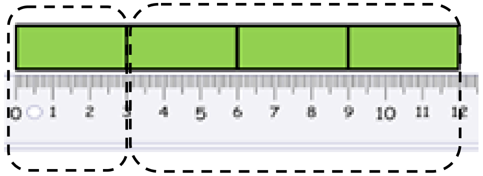


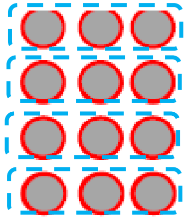
5 - 3 is equal to 3 - 1

Multiplication and Division

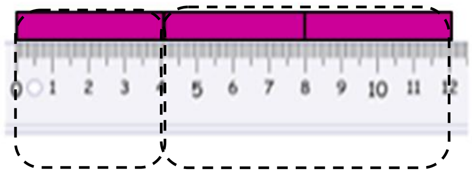
…continued

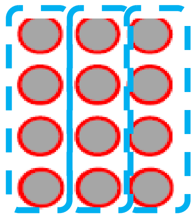
Leading to fractional understanding





I can see a quarter of 12 is equal to 3 and three quarters of 12 is equal to 9.





One third of 12 equals 3. Two thirds of 12 is 9.

Year 2
Numbers to 100

PA Plus

Herts for Learning

Key Stage 1 Examples

‘Think 10’ Regroup

5 + 6   7 + 4   9 + 7   7 + 6   8 + 7   7 + 5
2 + 18   4 + 18   8 + 19   47 + 6   68 + 7   9 + 87
13 - 8   27 - 8   53 - 6   68 - □ = 7   73 + □ = 89

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper

8 + 6 = □   5 + 7 = □   12 - 7 = □   46 + 7 = □
8 + 5 + 4 = □   55 + 17 = □   71 - 14 = □
86 - 21 = □   65 + □ = 93

‘Think Addition’ for subtraction

8 - 5   9 - 6   6 - 2   80 - 50   19 - 6   60 - 20

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper

10 - □ = 2   □ + 5 = 9   12 - 7 = □   19 - 9 = □
17 - 6 = □   39 - 8 = □   50 - □ = 20   56 - □ = 51

Reordering and finding complements

5 + 4 + 5   2 + 3 + 8   2 + 4 + 6   6 + 3 + 7
36 + 5 + 4   54 + 26

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper

4 + 5 + 6 = □   36 + 24 = □   69 + 11 = □

Compensation

2 + 9   12 + 9   9 + 72   2 + 19   19 + 42   42 + 39
5 + 8   15 + 8   65 + 8   18 + 5   55 + 18   48 + 35
12 - 9   22 - 9   52 - 9   52 - 19   92 - 19   92 - 39
12 - 8   22 - 8   52 - 8   52 - 18   92 - 18   92 - 48
48 + □ = 92   8 + □ = 52

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper

8 + 6 = □   52 + 7 = □   28 + □ = 35   69 + 11 = □
55 + 17 = □   39 - 8 = □   43 + 38 = □   70 - 18 = □

Rebalancing - Equal sum

12 + 9   9 + 72   24 + 19   15 + 42   44 + 37
5 + 8   15 + 8   65 + 7   18 + 6   55 + 15
48 + 35

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper

8 + 6 = □   4 + 5 + 6 = □   69 + 11 = □
55 + 17 = □   36 + 24 = □   43 + 38 = □
8 + 5 + 4 = □

Rebalancing - Equal difference

32 - 7   25 - 8   55 - 7   55 - 17   92 - 19
97 - 43   48 + □ = 92   8 + □ = 55

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper

12 - 7 = □   28 + □ = 35   71 - 14 = □
39 - 8 = □   86 - 21 = □   70 - 18 = □
65 + □ = 93

Reordering and multi-strategy

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper

8 + 5 + 4 = □

Double and near double facts

3 + 3   30 + 30   32 + 32   3 + 4   30 + 40
6 - 3   60 - 30   64 - 32   7 - 3   70 - 40
6 - □ = 3   60 - □□ = 30   64 - □□ =
□□ - 3 = 64

Find two ways of solving this: 70 - □0 = □0

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper

12 - 7 = □   □ + 5 = 9   50 - □ = 20

Make links to doubling and halving

3 + 3 = 2 x 3   2 x 30   2 x 3 + 1
6 ÷ 2   60 ÷ 2

Ensure that pupils can halve odd multiples of ten
50 ÷ 2 =

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper

3 x 2 = □   2 x 0 = □
8 ÷ 2 = □   ½ of 16 = □   ½ of 30 = □
12 ÷ 2 = □

Key facts	
Year One Recall • Number bonds within 10 including a + b + c = d, the effect of adding zero and missing number calculations • Reordering to find tens and some more e.g. 4 + 5 + 6 = • Doubles within 10 including subtraction e.g. 6 - 3 = 3 and missing numbers e.g. 6 - ☐ = 3 • Structured subitisation on tens frame to 20	**Year Two Recall** • Addition and subtraction facts to 20 • Multiplication and division facts 2, 5 and 10 x tables • Multiplication facts for 3 x tables • Number of minutes in an hour; number of hours in a day • Coin recognition up to £2 • Doubles to 20

PA Plus

Herts for Learning

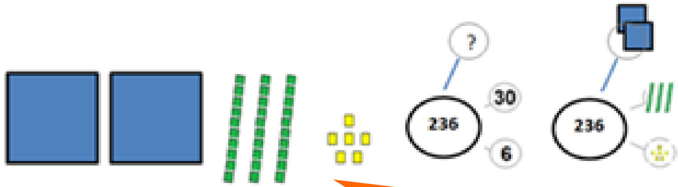
Year 3

Number and Place Value

Core concept: UNITISING and PLACE VALUE

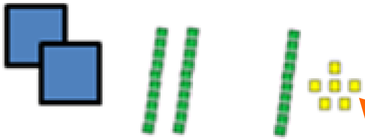
Core skill: REGROUPING

Grouping hundreds, tens and ones drawing out the concept that ten ones are equal to a unit of one ten and that ten tens are equal to a unit of one hundred.



I can see 2 hundreds, 3 tens and 6 ones 200 + 30 + 6.
236 is also six more than two hundred and thirty.

Using part part whole models, regroup 3-digit integers flexibly and in multiple ways.



236 can be regrouped into 220 and 16.
There are 23 tens and 6 ones in 236.

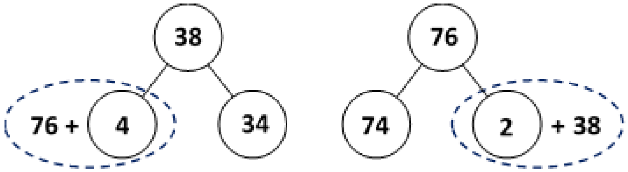
Addition and Subtraction

Core concept: CONSERVATION

Core skill: REGROUPING

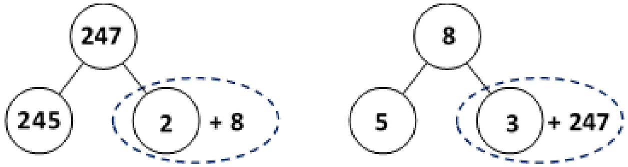
Think regroup for addition
Using part whole models draw out the skill of regrouping numbers to allow bridging through multiples of ten and a hundred. Ask pupils to reason why they may wish to reorder the numbers.

Pupils should be encouraged to explore multiple ways of regrouping both addends (refer to number and place value experiences). Only a limited example is shown here e.g. 76 + 38



76 + 38 = 80 + 34   76 + 38 = 74 + 40

This can be applied to regrouping addends in 3-digit + 1-digit calculations e.g. 247 + 8.



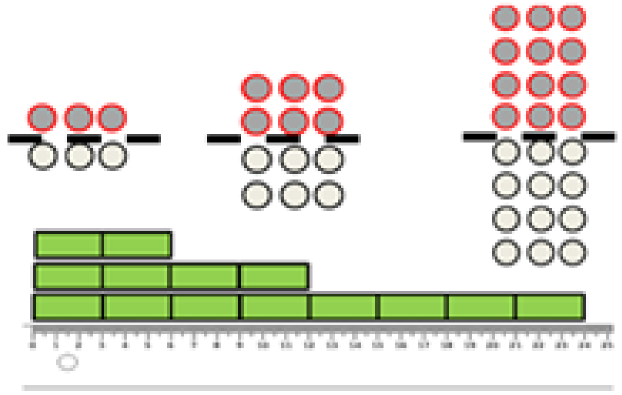
247 + 8 = 245 + 10   247 + 8 = 250 + 5

Multiplication and Division

Core concept: UNITISING and SCALING

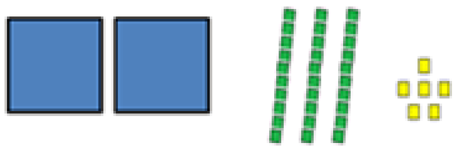
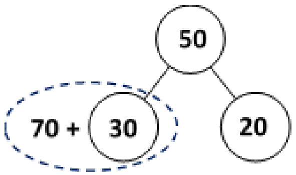
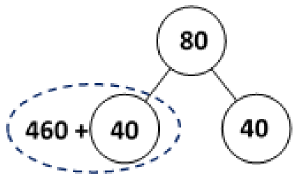
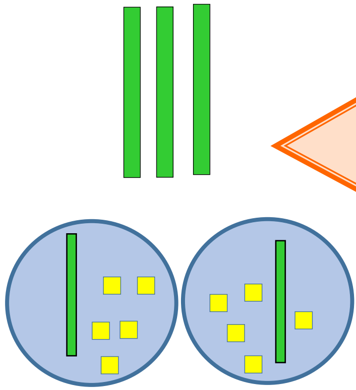
Core skill: DOUBLING and HALVING

To include ‘double and double’ strategy for x8 and halving strategy for finding x5.



I can find 5 lots by finding 10 lots and halving the product.

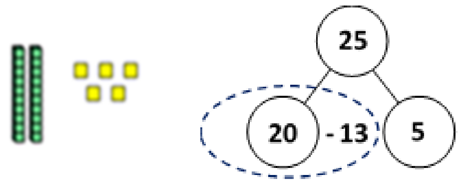
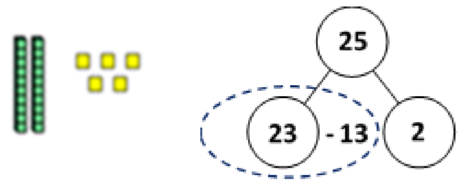
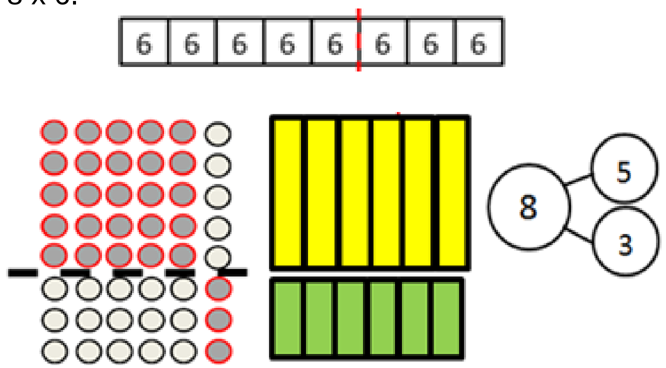
Year 3
Numbers to 1000

Number and Place Value	Addition and Subtraction	Multiplication and Division
Repeat this understanding to explore tenths through the same concrete and pictorial representations securing the multiplicative relationship.  3 ones and 4 tenths.  2 tens, 3 ones and 6 tenths.	This can also be applied to regroup to bridge through multiples of 100 e.g. 70 + 50 or 460 + 80.  70 + 50 = 100 + 20  460 + 80 = 500 + 40 This sum is easier if I regroup the 80 into 40 and add it to the 460 to make 500.	Ensure pupils can double and halve 2-digit numbers and generalise what happens when we halve a number with an odd multiple of tens.  If I halve 30, I can share one ten into each group but then I have to regroup the last ten into ten ones. Each group will then get five ones. So 1 ten and 5 ones in each group is 15. Extend to halving odd multiples of 100 and later odd ones. Pupils should also be able to use doubles knowledge to solve near double questions e.g. 70 + 60. 70 + 60 is like double 60 plus 10. It's also 10 less than double 70.

Year 3
Numbers to 1000

PA Plus

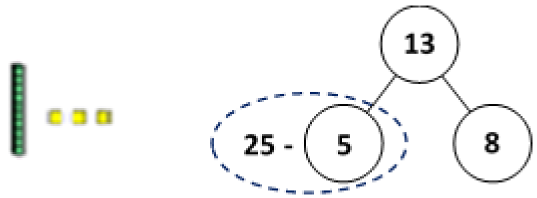
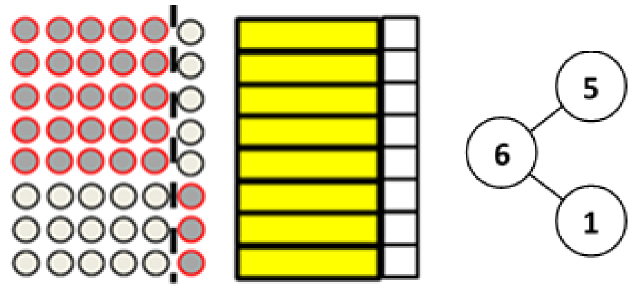
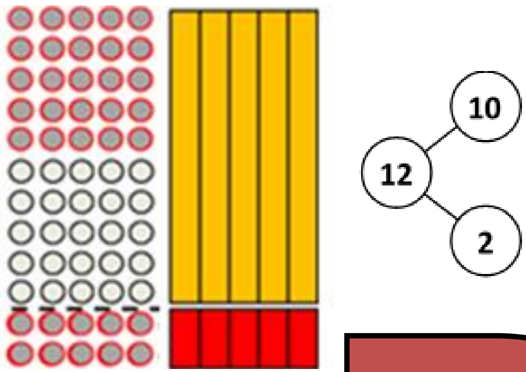
Herts for Learning

Number and Place Value	Addition and Subtraction	Multiplication and Division
	Core concept: UNITISING **Core skill: REGROUPING** **Think Regroup for subtraction** drawing out the skill of regrouping either the minuend or subtrahend including when bridging through multiples of ten and a hundred e.g. 25 - 13. **Regrouping the minuend** I regrouped the 25 into 20 and 5. Then I took the 13 from the 20 and that left 7. Now I add that to the 5 which makes 12.   I regrouped the 25 into 23 and 2. Then I took the 13 from the 23 and that left 10. Now I add that to the 2 which makes 12.	**Core concept: CONSERVATION** **Core skill: REGROUPING** **Think 5x fact** Application of the distributive law. Regrouping the multiplier (number of groups) e.g. 8 x 6.  8 x 6 = 5 x 6 + 3 x 6 I can use my 5 fact to solve 8 x 6. I know 5 x 6 is 30 and 3 x 6 is 18. So 8 x 6 is 48.

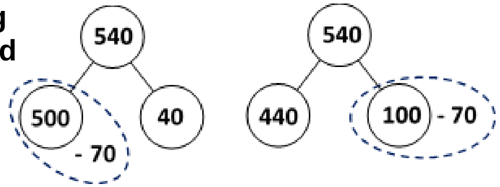
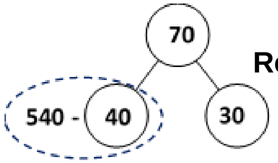
Year 3
Numbers to 1000

PA Plus

Herts for Learning

Number and Place Value	Addition and Subtraction	Multiplication and Division
	Regrouping the subtrahend  I regrouped the 13 into 5 and 8. Then I took the 5 from the 25 and that left 20. Now I can take away the remaining 8 to make 12.	Regrouping the multiplicand (number in the group) e.g. 8 x 6.  8 x 6 = 5 x 8 + 1 x 8 I can use my 5 fact to solve 8 x 6. I know 8 x 5 is 40 and 8 x 1 is 8. So 8 x 6 is 48. Applying to **think 10**. For example, 12 x 5.  12 x 5 = 10 x 5 + 2 x 5

Year 3
Numbers to 1000

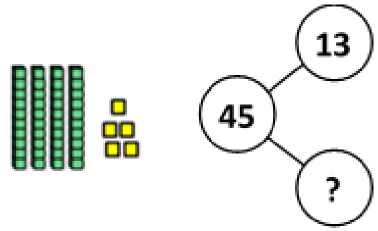
Number and Place Value	Addition and Subtraction	Multiplication and Division
	Then applied to HTO - O and HTO - TO. For example, 540 - 70. **Regrouping the minuend**   **Regrouping the subtrahend** Apply to contexts of measures such as money and time e.g. £3 and 40p subtract 60p I can regroup the 60p into 40p and 20p. First, I take the 40p away. That gets me to £3. Next, I take the 20p away, which is £2 and 80p. **Core concept: UNITISING** **Core skill: FINDING COMPLEMENTS / REORDERING** **Reordering and finding complements** Adding three or more numbers. Draw out reasons why children may wish to reorder the numbers. Focus upon the range of strategies used. 6 + 9 + 4 + 5 + 1 = 75 + 95 + 25 = 1.5 + 3 + 0.5 =	

Number and Place Value	Addition and Subtraction	Multiplication and Division

Addition and Subtraction

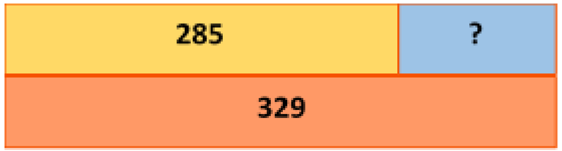
Core skill: APPLYING THE INVERSE

Think addition for subtraction



45 - □ = 13
13 + □ = 45

Thirteen and what makes 45?



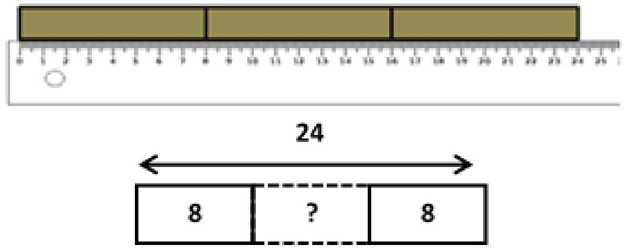
329 - 285 = □ so 285 + □ = 329

Multiplication and Division

Core skill: APPLYING THE INVERSE

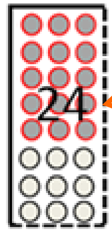
Think multiplication for division e.g. 24 ÷ 8

By grouping

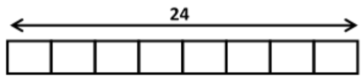


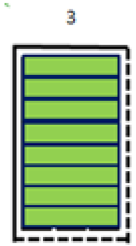
I can see that 3 groups of 8 equal 24.
3 x 8 = 24, 24 ÷ 8 = 3

By sharing



I can see that 8 groups of 3 equal 24.
8 x 3 = 24, 24 ÷ 8 = 3





Leading to fractional understanding – see Year 2 progression and extend into further fractions.

Year 3
Numbers to 1000

PA Plus

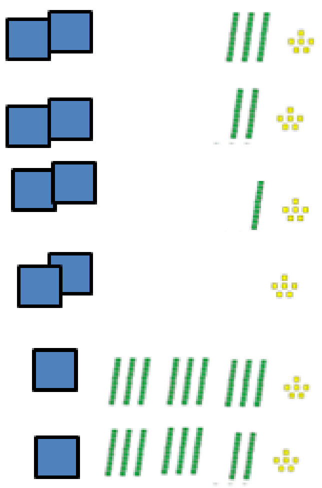
Herts for Learning

Number and Place Value

Core concept: COUNTING and PLACE VALUE

Core skill: COUNTING ON and BACK

Counting on and back in tens and hundreds from any number allowing children to notice in the pattern what changes as a result (and what doesn't change).



236 2 hundreds, 3 tens and 6 ones

226 2 hundreds, 2 tens and 6 ones

216 2 hundreds, 1 ten and 6 ones

206 2 hundreds, 0 tens and 6 ones

196 1 hundred, 9 tens and 6 ones

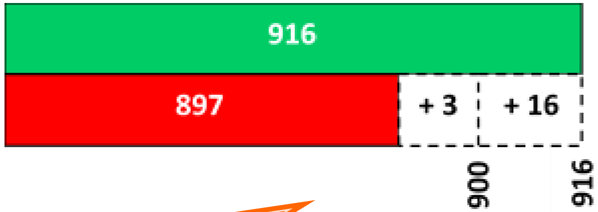
186 1 hundred, 8 tens and 6 ones

Addition and Subtraction

Core concept: COMPARISON

Core skill: COUNTING ON and BACK

Count on to find the difference drawing out the use of complements to benchmark numbers e.g. 916 – 897.

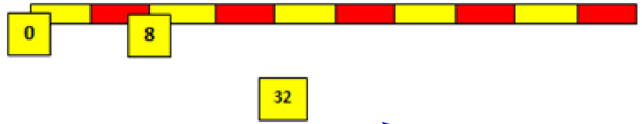


There is 3 more to 900 and 16 more to 916.
3 + 16 = 19

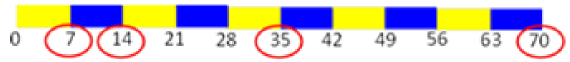
Multiplication and Division

Core concept: UNITISING and SCALING

Core skill: COUNTING ON and BACK



Where on the stick will we place 32? Explain how you know.



If I know x1, x2, x5, x10, what else can I work out?

1 more, 1 less → x4 x6 x9 x3

2 more, 2 less → x7 x8 x4 x3

Year 3
Numbers to 1000

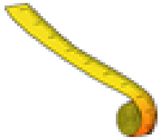
PA Plus

Herts for Learning

Number and Place Value	Addition and Subtraction	Multiplication and Division

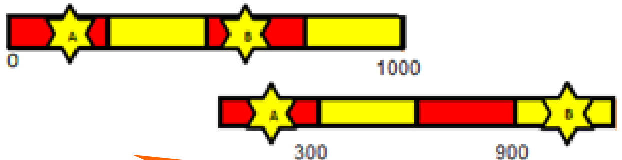
Number and Place Value

Core concept: MAGNITUDE and COMPARISON



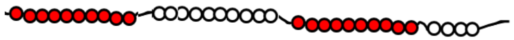
I can find 236cm on my tape measure. It is 36cm greater than 200cm and 64cm less than 300cm.

Number magnitude drawing out the concepts of relative size, order and comparison of number.



I think that number A is 200 because it is nearly 250.

Estimation drawing out concepts of distance of numbers to target numbers to prepare for rounding.

200 and 

234 is 4 from 230 and 6 from 240.
234 is nearer to 230 than to 240.

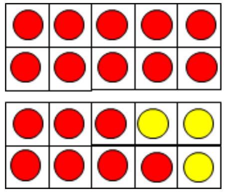
Addition and Subtraction

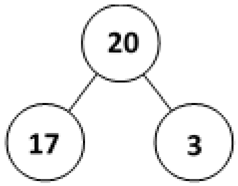
Core concept: COMPARISON

Core skill: COMPENSATION

Part whole model

45 - 17





45 - 17

= 45 - 20 + 3
= 25 + 3
= 28

Subtracting 17 is the same as subtracting 20 and adding 3 back.

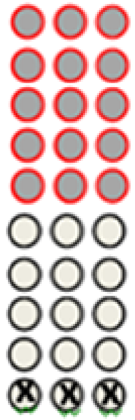
Multiplication and Division

Core concept: COMPARISON and SCALING

Core skill: COMPENSATION

9 x 3 = 10 x 3 - 3

9 x 3 = 10 x 3 - 1 x 3



Nine groups of three is equal to 10 groups of three, less 1 group of 3.

Year 3
Numbers to 1000

PA Plus

Herts for Learning

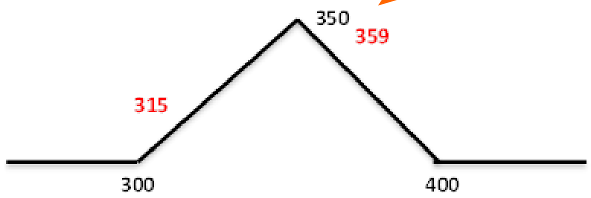
Number and Place Value	Addition and Subtraction	Multiplication and Division

Number and Place Value

Core skill: ROUNDING

Leading to rounding to the nearest 10, the nearest 100 and, later, the nearest 1.

359 is nearer to 400 than 300 so it rounds to 400.



315 is nearer to 300 than 400 so it rounds to 300.

Addition and Subtraction

Core concept: COMPARISON

Core skill: COMPENSATION

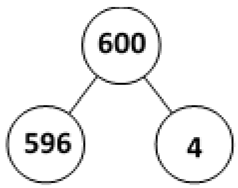
£3 and 40p add 80p

£3 and 40p

£3.40 + £1 - 20p
= £4.40 - 20p
= £4.20

Adding 80p is the same as adding £1 and subtracting 20p.

632 - 596



Subtracting 596 is the same as subtracting 600 and adding 4 back.

Multiplication and Division

Year 3
Numbers to 1000

PA Plus

Herts for Learning

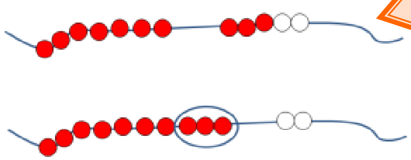
Number and Place Value	Addition and Subtraction	Multiplication and Division

Addition and Subtraction

Core concept: CONSERVATION and SUM

Core skill: REBALANCING

Equal sum drawing out the idea of equality when we rebalance the numbers in an addition calculation e.g. 7 + 5.



I can move 3 beads from the 5 to the 7 and the sum will stay the same. Now I have 10 + 2.

e.g. 52 + 37

52 + 37

I move 2 beads from the 52 and give them to the 37.
Now I can solve 50 + 39. It's easier.

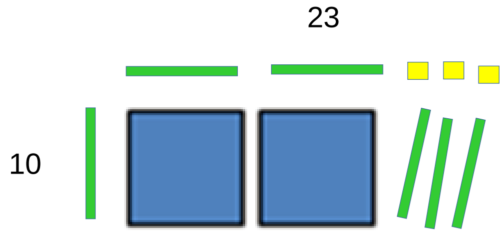
Multiplication and Division

Core concept: PLACE VALUE and SCALING

Core skill: x ÷ BY 10

Check pupils understand the concept of multiplying and dividing by 1 and 0.

Place value drawing out the implications of multiplying and dividing by ten and 100 on 2-digit numbers e.g. 23 x 10.



23 groups of ten.
20 groups of ten is equal to 200.
3 groups of ten is equal to 30.
23 groups of 10 is equal to 230.

Year 3
Numbers to 1000

PA Plus

Herts for Learning

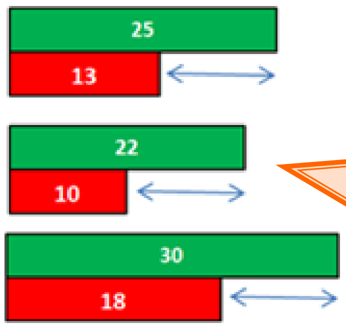
Number and Place Value

Addition and Subtraction

Core concept: COMPARISON

Core skill: REBALANCING

Equal difference using comparison drawing out the concept that adding or subtracting the same quantity from both the subtrahend and minuend maintains the difference between the numbers.



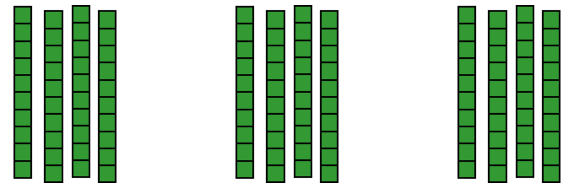
I can take 3 from each number and the difference will remain equal.

I can add 5 to each number and the difference will remain equal.

If I wanted to solve 21 - 16, I can take 1 from each number and solve it as 20 - 15. That is an easier calculation.

Multiplication and Division

$\square = 3 \times 40$



3 groups of 4 tens is equal to 12 tens.
12 tens is 120.
So 120 = 3 x 40

$\square = 120 \div 3$

120 is 12 tens.
12 tens divided into 3 groups is equal to 4 tens. 4 tens is 40. So 40 = 120 ÷ 3.

Year 3
Numbers to 1000

PA Plus

Herts for Learning

Year 4

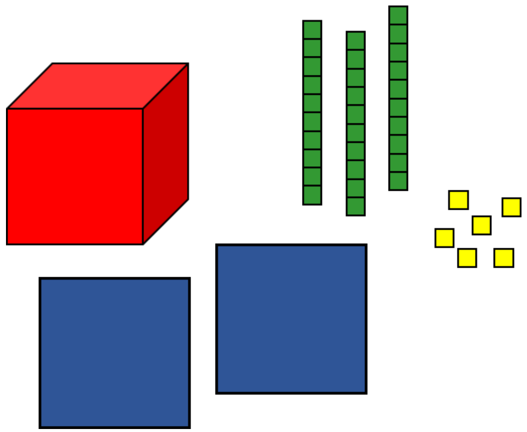
Number and Place Value	Addition and Subtraction	Multiplication and Division

Number and Place Value

Core concept: UNITISING

Core skill: REGROUPING

Grouping thousands, hundreds, tens and ones drawing out the concept that ten ones are equal to a unit of 'one ten' and ten tens are equal to a unit of one hundred etc.



I can see one thousand, two hundreds, three tens and six ones.
1000 + 200 + 30 + 6
It is thirty-six more than one thousand and two hundred.

Addition and Subtraction

Core concept: UNITISING

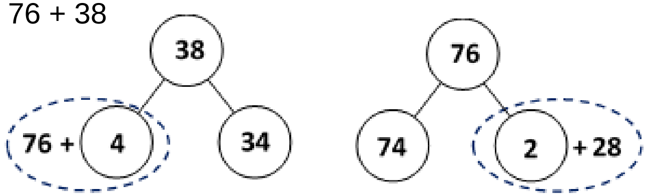
Core skill: REGROUPING

Think Regroup for addition

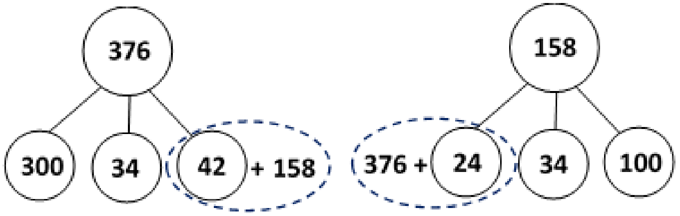
Part whole drawing out the concept of regrouping numbers to allow bridging through hundreds, tens and ones. Ask pupils to reason why they may wish to reorder the numbers.

Pupils should continue Year 3 learning and be encouraged to explore multiple ways of regrouping both addends (refer to number and place value experiences). Only a limited example is shown here.

76 + 38



This can be adapted to 'Think 100'
376 + 158



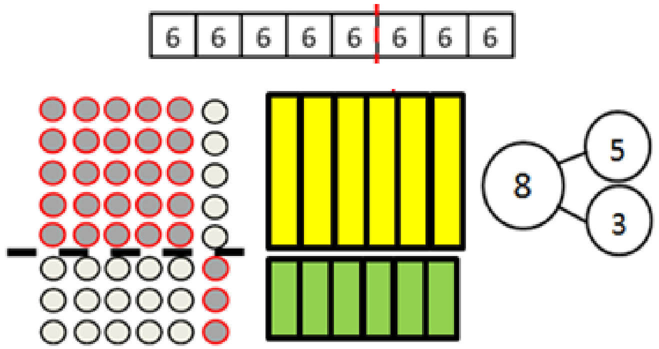
Multiplication and Division

Core concept: CONSERVATION and SCALING

Core skill: REGROUPING

Think 5x fact
Application of the distributive law

Regrouping the multiplier (number of groups).
For example, 8 x 6.



I can use my 5 fact to solve 8 x 6.
I know 5 x 6 is 30. 3 x 6 is 18.
So 8 x 6 is 48.

8 x 6 = 5 x 6 + 3 x 6

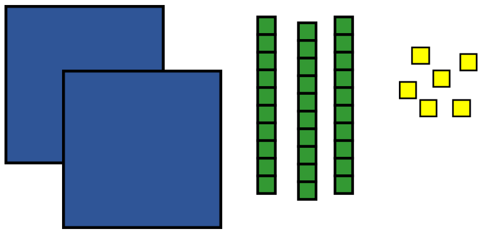
Year 4
Numbers to 10000

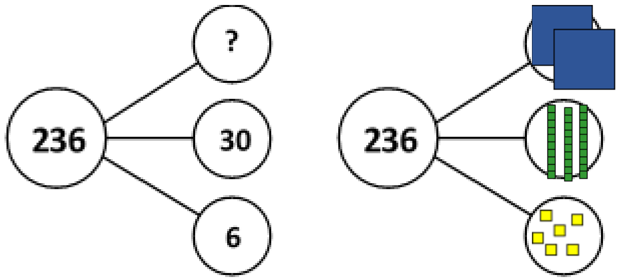
PA Plus

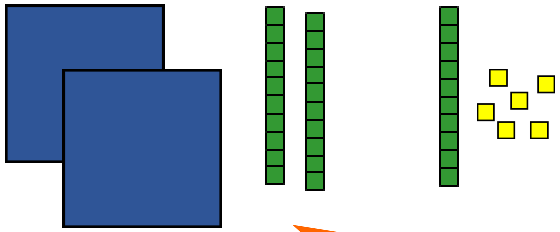
Herts for Learning

Number and Place Value

Part part whole
Regroup 3-digit and 4-digit integers flexibly and in multiple ways.







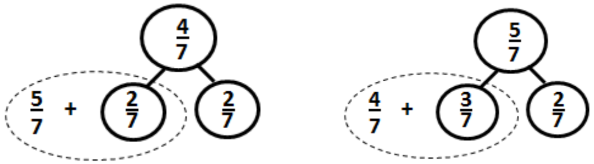
236 can be regrouped into 220 and 16.

Addition and Subtraction

Adapt '**Think Regroup'** strategy to decimal and fractional part whole as well as measures such as time and money.

For example: $\frac{4}{7} + \frac{5}{7} =$

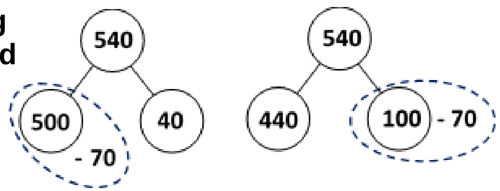
Both addends can be regrouped using complements to 1 and '*some more'*.

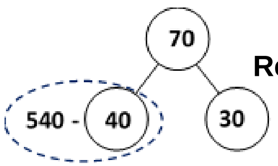


Think Regroup for subtraction

Part whole drawing out the skill of regrouping either the minuend or the subtrahend.
Pupils should be encouraged to explore multiple ways of regrouping both the minuend and subtrahend (refer to number and place value experiences).
For example, 540 - 70.

Regrouping the minuend

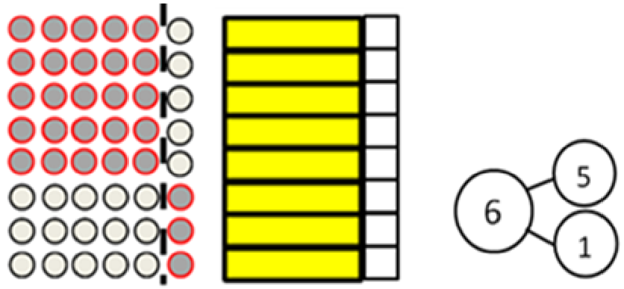




Regrouping the subtrahend

Multiplication and Division

Regrouping the multiplicand (number in the group) e.g. 8 x 6.



I can use my 5 fact to solve 8 x 6.
I know 8 x 5 is 40. 8 x 1 is 8.
So 8 x 6 is 48.

8 x 6 = 8 x 5 + 8 x 1

Regrouping used for multiple strategies.

7 x 8

I can find 7 x 8 in lots of ways if I didn't know it.
I can try 7 x 4 x 2 or 8 x 8 - 8.

Year 4
Numbers to 10000

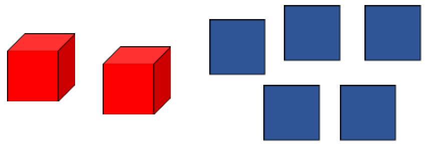
PA Plus

Herts for Learning

Number and Place Value	Addition and Subtraction	Multiplication and Division

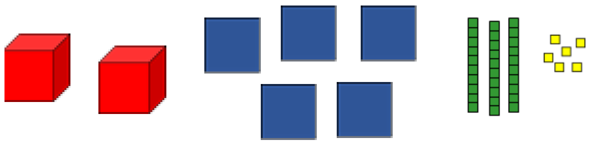
Number and Place Value

Use resources to prove statements such as:
'There are 25 hundreds in the number 2500'.



Continue this to explore tenths and hundredths through the same concrete and pictorial representations to secure understanding of the multiplicative relationship.

I can see three tenths and four hundredths.



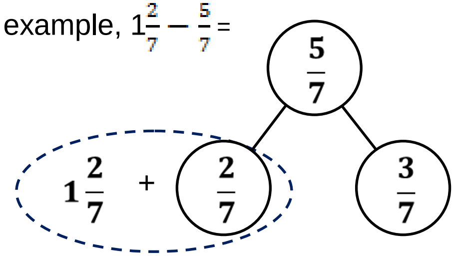
I can see that in the number 25.36, there are 2 tens, 5 ones, 3 tenths and 6 hundredths.

We can also say there are 253 tenths and 6 hundredths in my number.

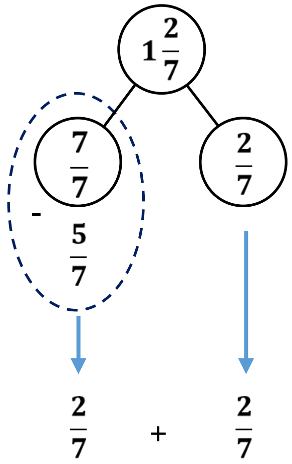
Addition and Subtraction

Then adapted to decimal and fractional part whole as well as measures such as time and money.

For example, $1\frac{2}{7} - \frac{5}{7} =$



I can regroup the subtrahend $\frac{5}{7}$ into $\frac{2}{7}$ and $\frac{3}{7}$. Then I can take away the $\frac{2}{7}$ leaving $\frac{7}{7}$ or 1 and finally take away $\frac{3}{7}$.



... or I could regroup the minuend, subtract from the 1 and then recombine.

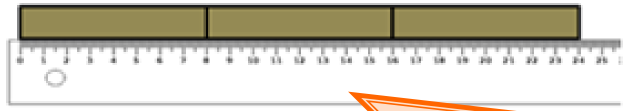
Multiplication and Division

Core skill: APPLYING THE INVERSE

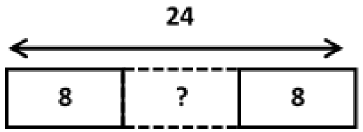
Think multiplication for division

For example, 24 ÷ 8.

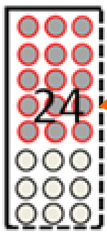
By grouping



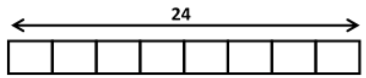
I can see that 3 groups of 8 equal 24.
3 x 8 = 24, 24 ÷ 8 = 3

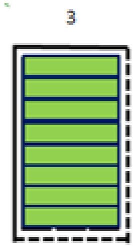


By sharing



I can see that 8 groups of 3 equal 24.
8 x 3 = 24, 24 ÷ 8 = 3





Leading to fractional understanding – see Year 2 progression and extend into further fractions.

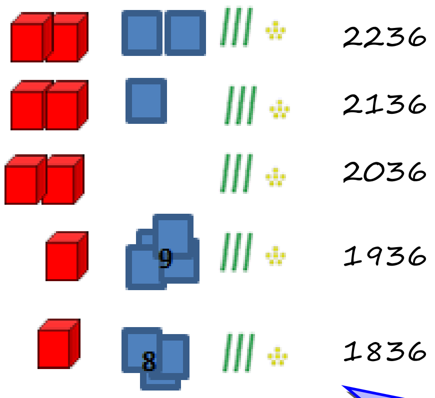
Year 4
Numbers to 10000

Number and Place Value

Core concept: COMPARISON

Core skill: COUNTING ON and BACK

Counting on and back in hundreds and tens from any number. Allow pupils to notice the pattern. What changes as a result and what does not.



Show me the difference between 2036 and 2136.

Counting on and back in multiples as well as making counting links e.g. counting in 6s, 60s (relate to time), 600s and 0.6s.

Counting in 25s, 50s, 0.1s and 0.01s paying attention to bridging (regrouping) points.

Addition and Subtraction

Core concept: COMMUTATIVTY and UNITISING

Core skill: REORDERING and FINDING COMPLEMENTS

Adding three or more numbers. Draw out the reasons why pupils may wish to reorder the numbers and focus on a range of strategies used.

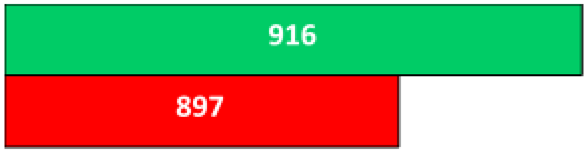
800 + 240 + 360 =
2.5 + 25 + 5 + 2.5 =
310 + 700 + 300 =

Core concept: COMPARISON and DIFFERENCE

Core skill: COUNTING ON and BACK

916 – 897

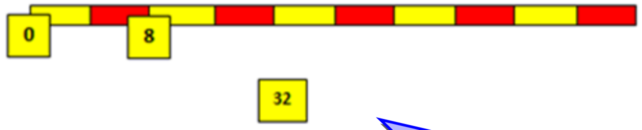
Comparison
Drawing out use of benchmark numbers.



Multiplication and Division

Core concept: UNITISING and SCALING

Core skill: COUNTING ON and BACK



Where on the counting stick will we place 32? Explain your thinking.

If I know x1, x2, x5, x10, what else can I work out?

1 more, 1 less: x4 x6 x9 x3

2 more, 2 less: x7 x8 x4 x3

Year 4
Numbers to 10000

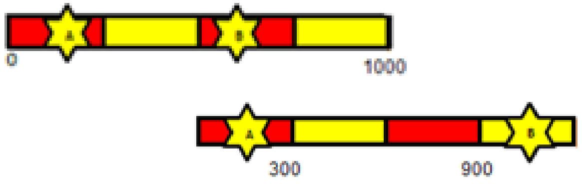
PA Plus

Herts for Learning

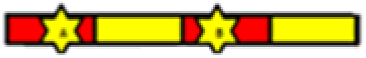
Number and Place Value

Core concept: MAGNITUDE

Number magnitude drawing out the concepts of relative size, order and comparison of number.



The number couldn't be…because ….
It could be … because…



0.5

Number estimation (using scales) should be applied to different scales of measures. This should include those with negative and dial scales.

Core skill: ROUNDING

Leading to rounding to the nearest 10, 100, 1000, hour and £1 etc.

Addition and Subtraction

Core concept: COMPARISON, SUM and DIFFERENCE

Core skill: COMPENSATION and REBALANCE

Equal sum drawing out the concept of equality when rebalancing the numbers in an addition calculation.

255 + 49 is easier if I take one from the 255 and give it to the 49. My sum stays equal. Then my sum becomes 254 + 50 = 304.

Compensation with the same calculation supports pupils' multi-strategy approach. Pupils can begin to evaluate strategies.

I could also think of 255 + 49 as compensation because adding 49 is like adding 50 and taking one away. Now my sum looks like this: 255 + 50 - 1 = 304.

Multiplication and Division

Core concept: COMPARISON and SCALING

Core skill: COMPENSATION

3 x 9 = 3 x 10 - 3

Nine groups of three is equal to ten groups of three, less 1 group of 3.

I could use this to find 90 x 3. 100 x 3 - 10 x 3

Year 4
Numbers to 10000

PA Plus

Herts for Learning

Number and Place Value	Addition and Subtraction	Multiplication and Division

Addition and Subtraction

Rebalancing

It is 1.44pm. What time will it be in 50 minutes?

10 minutes

1 hour 44 minutes + 50 minutes =
1 hour 34 minutes + 60 minutes =
2 hours and 34 minutes =
2.34pm

If I give 10 minutes from the 1 hour 44 to the 50 minutes then I can add on 1 hour.

Compensation

It is 1.44pm. What time will it be in 50 minutes?

Adding 50 minutes is like adding one hour and taking away 10 minutes.

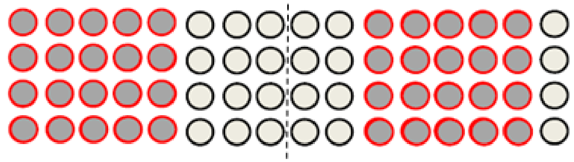
Multiplication and Division

Core concept: CONSERVATION and SCALING

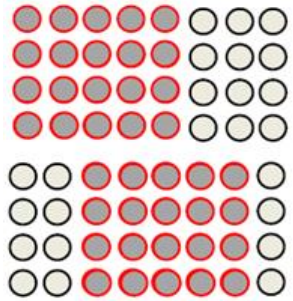
Core skill: REARRANGING

Children should explore **rearranging** arrays to simplify multiplications but conserve the area.

4 x 16 =



Can become 8 x 8

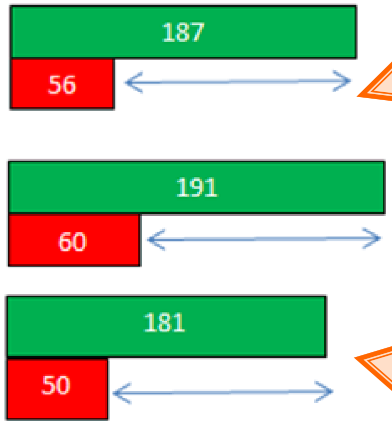
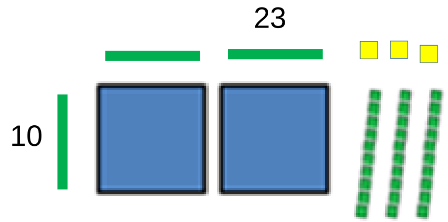


Doubling the 4x and halving the group of 16 gives me 8 x 8. The area remains equal.

Year 4
Numbers to 10000

PA Plus

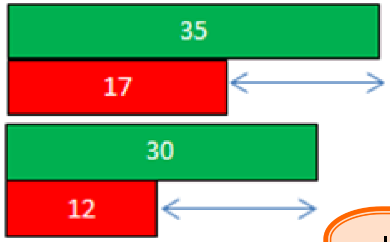
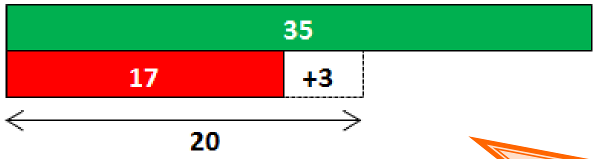
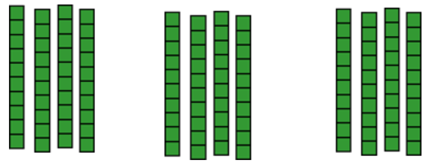
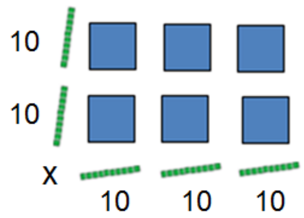
Herts for Learning

Number and Place Value	Addition and Subtraction	Multiplication and Division
	Equal difference using comparison drawing out the concept that adding or subtracting the same quantity from both the subtrahend and minuend maintains the difference between the numbers.  I can add 4 to each number and the difference will remain equal. I can take 6 from each number and the difference will remain equal. Which strategy did you prefer? Explain why.	**Core concept: PLACE VALUE** **Core skill: x ÷ BY 10, 100** **Check pupils understand the concept of multiplying and dividing by 1 and 0.** **Place value** drawing out the implications of multiplying and dividing by 10 and 100 on 2-digit numbers e.g. 23 x 10.  23 groups of ten. 20 groups of ten is equal to 200. 3 groups of ten is equal to 30. 23 groups of 10 is equal to 230.

Year 4
Numbers to 10000

PA Plus

Herts for Learning

Number and Place Value	Addition and Subtraction	Multiplication and Division
	I had £35. I bought a book for £17. How much change do I get?  I can take 5 from each number and the difference will remain equal. Compare this to **compensation**.  Subtracting £17.00 is like subtracting £20.00 and adding back £3.00.	☐ = 3 x 40  3 groups of 4 tens is equal to 12 tens. 12 tens is 120. So 120 = 3 x 40. ☐ = 120 ÷ 3 120 is 12 tens. 12 tens divided into 3 groups is equal to 4 tens. 4 tens is 40. So 40 = 120 ÷ 3. 20 x 30 = ☐  When I multiply 10 by 10 it equals 100. 2 tens multiplied by 3 tens is equal to 6 hundreds.

Year 4
Numbers to 10000

Lower KS2 examples

'Think Regroup' for addition

Think 10

37 + 45 68 + 23 29 + 75
76 + 27 55 + 16 42 + 38
537 + 8 727 + 5 213 + 18 146 + 37
36 - □ = 29 56 - 2□ = 33 □7 – 45 = 32

Think 100

290 + 13 370 + 50 580 + 73
270 + 51 67 + 350 860 + 69
86 + 770 680 + 63

Think 1000

4900 + 500 4800 + 260 6900 + 430
3200 + 910 230 + 7900 570 + 8500
3700 + 370 3622 + 500

Think 1

2.7 + 1.4 $2\frac{8}{10} + \frac{3}{10} =$ 6.5 + 5.6
$1\frac{7}{8} + 1\frac{5}{8}$

'Think Regroup' for subtraction

Think 10

97 - 8 74 - 7 53 - 5 63 - 37
77 - 32 84 - 26 57 - 28 256 - 37
25 + □= 85 163 + □ = 363 426 + 2□2 = 668

Think 100

230 - 70 660 - 82 420 - 77 950 - 147

Think 1

1.3 - 0.6 $1\frac{4}{8} - 1\frac{5}{8}$ 3.4 - 2.7 $2\frac{1}{3} - 1\frac{2}{3}$

Re-ordering and finding complements

Complements to 10

8 + 6 + 2 + 3 + 4 3 + 5 + 7 + 5 +4
1 + 4 + 6 +7 + 9 30 + 50 + 70
25 + 50 + 5 75 + 40 + 20 + 25

Complements to 100

400 + 547 + 600 700 + 240 + 300
750 + 400 + 250

Complements to 1

2.7 + 4 + 1.3 4.6 + 5 + 2.4 8.2 + 3 + 5.8

Compensation

42 + 29 45 + 27 24 + 47 28 + 65 68 + 27
232 + 49 856 + 17 48 + 325 232 + 95
132 + 59 568 + 195 399 + 423 412 + 298
405 + 199 597 + 308
43 - 18 94 - 37 54 - 29 77 - 9 82 - 23
483 - 99 256 - 98 398 - 74 597 - 63
401 - 97 736 - 301 613 - 299 743 - 397
298 - 156 799 - 403

Rebalancing - Equal sum

45 + 27 26 + 39 78 + 18 65 + 27
73 + 39 84 + 47 42 + 97 116 + 35
368 + 123 404 + 198 356 + 427 528 +298
3.7 + 1.9 7.6 + 4.7 1.9 + 5.8

Rebalancing - Equal difference

75 - 28 56 - 29 78 - 38 55 - 27
83 - 21 75 - 12 95 - 42 67 - 51
912 - 797 837 - 498 711 - 467 628 -198
482 - 302 729 - 404 548 - 202 637- 203
6.4 - 3.9 6.6 - 3.2 7.7 - 4.8 $1\frac{2}{7} - \frac{5}{7}$

Counting on to subtract

315 - 298 412 - 396 917 - 898 611 - 598

Think multiplication

85 ÷ 5 72 ÷ 4 99 ÷ 6 240 ÷ 12
660 ÷ 3 210 ÷ 7 540 ÷ 9 500 ÷ 4
□ ÷ 3 = 8 3□ ÷ 5 = 6

X and ÷ 10, 100 and 1000

4 x 30 9 x 30 70 x 70 60 x 50
300 x 4 800 x 7 9 x 800 6 x 400
3 x 2000 4000 x 6 8 x 7000 9 x 8000
500 ÷ 10 400 ÷ 5 600 ÷ 2 240 ÷ 4
120 ÷ □ = 12 365cm = □ m 750mm = □ cm

Double and near double facts

7 x 20 3 x 38 9 x 200 11 x 4
16 x 20 18 x 2000
80 ÷ 4 160 ÷ 4 1600 ÷ 4 2400 ÷4

Think 5 / Think 10 for multiplication

28 x 5 16 x 8 23 x 9 92 x 8 52 x 4
13 x 21 34 x 19 123 x 4 214 x 6 9 x 234
11 x 314 21 x 400 400 x 38

Key facts	
Year Three Recall • Sums and differences between pairs of numbers which are multiples of 10 and 100 • Doubles and halves of multiples of 10 or 100 • Complements to 100 • Complements to 60 (time) • Complements of tenths that make 1 • Complements of fractions with the same denominator that make 1 e.g. 3/7 + 4/7 = 1 • x 3, x 4, x 8 facts including division facts • Number of seconds in a minute • Number of days in a month and in a year including a leap year	**Year Four Recall** • Review addition and subtraction facts within 20, ensure application to 10, 100 and 1000 (6 + 3, 60 + 30, 600 + 300, 6000 + 3000) • Doubles and halves of multiples of 10, 100 or 1000 (6 + 6, 60 + 60, 600 + 600, 6000 + 6000) • All multiplication and division facts to 12 x 12 • Multiplication and division by zero and one facts • Division and multiplication by 10 and 100 • Conversion of kilometres to metres, hours to minutes, years to months, weeks to days • Complements of hundredths that make 1

PA Plus

Herts for Learning

Years 5 and 6

Number and Place Value

Core concept: MAGNITUDE and COMPARISON

Introduce pupils to a range of calculations in which a secure understanding of place value is required to support the solution.

For example, 2.005 + 3.24.

Estimation drawing out the concept of 'distance' of numbers to target numbers / benchmarks in preparation for rounding.

20,000 and 

I can see that 20,034 is 4 from 20,030 and 6 from 20,040.
20,034 is nearer to 20,030 than to 20,040.

Comparison to benchmark numbers
Using number knowledge to look for 'nearly numbers' in calculations.

7834 + 79,996

79,996 is 4 less than 80,000 and that's an easier number to add.

Addition and Subtraction

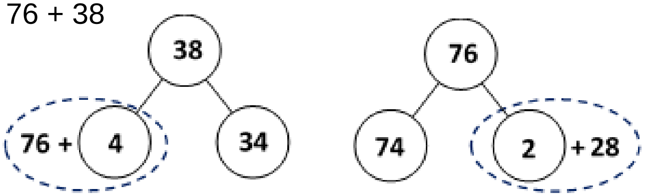
Core concept: UNITISING

Core skill: REGROUPING

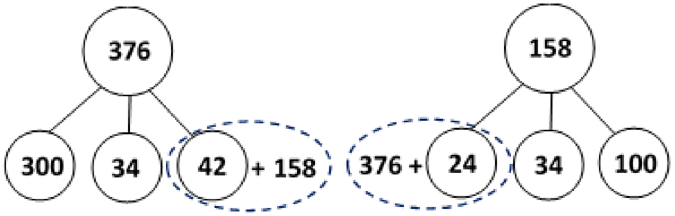
Think Regroup for addition
Part whole drawing out the skill of regrouping numbers to allow bridging through hundreds, tens and ones. Ask pupils to reason why they may wish to reorder the numbers.

Pupils should continue LKS2 learning and be encouraged to explore multiple ways of regrouping both addends (refer to number and place value experiences). Only a limited example is shown here.

76 + 38



376 + 158



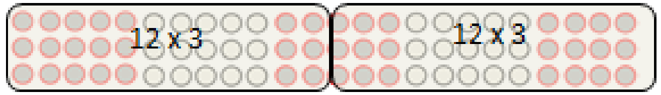
Multiplication and Division

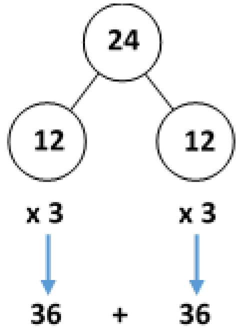
Core concept: UNITISING

Core skill: REGROUPING

Think Regroup for multiplication and division drawing out the distributive law for both multiplication and division and encouraging pupils to regroup and multiply in a variety of ways, evaluating the most useful.

24 x 3 =





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Number and Place Value	Addition and Subtraction	Multiplication and Division

Number and Place Value

Rounding

Round 136,521 to the nearest 100, 1000 and 10,000.

Rounding as estimation for multiplication and division.

688 x 79 =

688 rounds to 700 and 79 rounds to 80. The calculation 688 x 79 is close to 700 x 80, which is 56,000.

789 ÷ 79 =

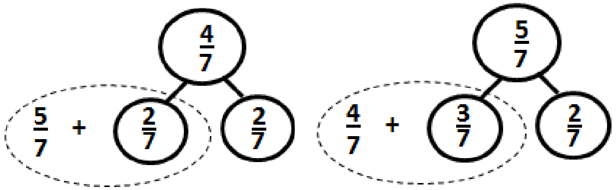
789 rounds to 800 and 79 rounds to 80. The calculation 789 ÷ 80 is close to 800 ÷ 80, which equals 10.

Addition and Subtraction

Then adapted to decimal and fractional part whole as well as measures such as time and money.

For example, $\frac{4}{7} + \frac{5}{7} =$

Here **both addends** can be regrouped using **complements to 1 and *some more.***

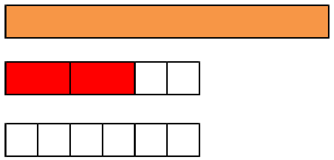


Extend into UKS2 by converting fractions into equivalents with common denominators. Beginning with conversions where no regrouping is required.

For example:

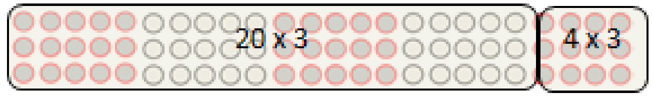
$\frac{2}{10} + \frac{2}{5} =$

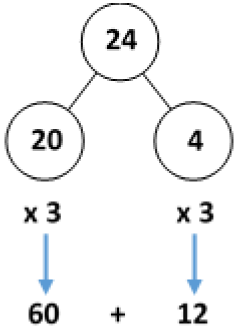
$\frac{2}{10} + \frac{4}{10} =$

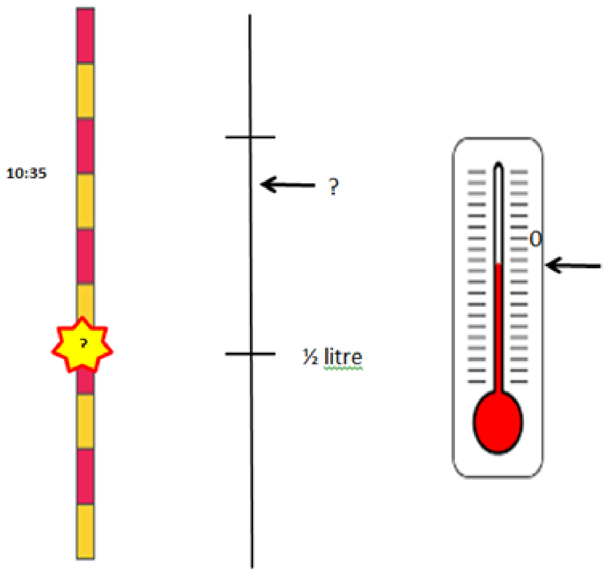
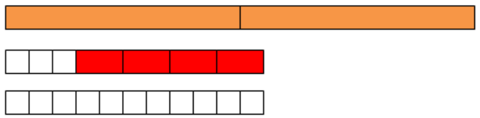
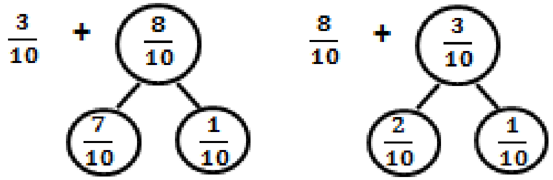
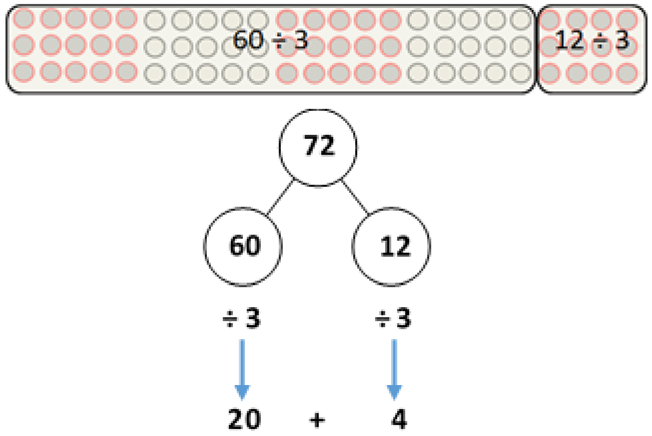


Multiplication and Division

24 x 3 =





Number and Place Value	Addition and Subtraction	Multiplication and Division
Number magnitude drawing out the concepts of relative size, order and comparison of number.  Number estimation using scales should continue to be applied to scales of measurement including those with negative and dial scales.  	Progress to examples where regrouping would be a valid strategy. $\frac{3}{10} + \frac{4}{5} =$ $\frac{3}{10} + \frac{8}{10} =$  Rehearse regrouping **either addend to make 1s and some more.**  	72 ÷ 3 =  15 x 3.4 = I know that 10 x 3.4 = 34 Then I can halve 34 to find 5 groups of 3.4 which is 17. After that, I have to recombine the products. This equals 51.

<table>
<tr><th>Number and Place Value</th><th>Addition and Subtraction</th><th>Multiplication and Division</th></tr>
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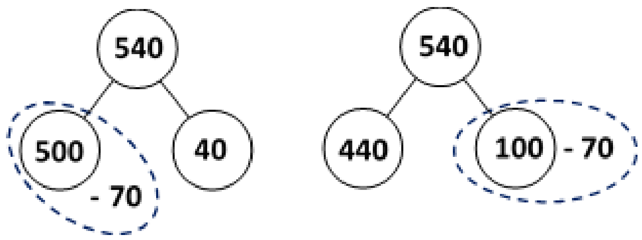
Think regroup for subtraction

Part whole drawing out the skill of regrouping **either the minuend or the subtrahend**.

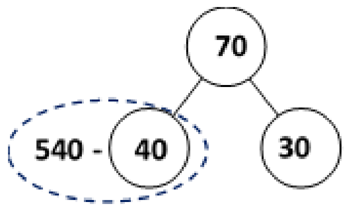
Pupils should be encouraged to explore multiple ways of regrouping both the **minuend** and **subtrahend** (refer to number and place value experiences).

For example, 540 – 70

Regrouping the minuend



Regrouping the subtrahend



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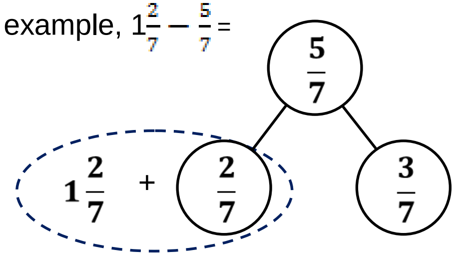
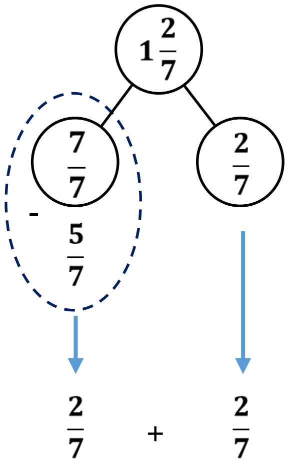
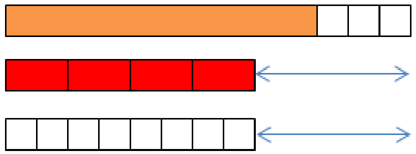
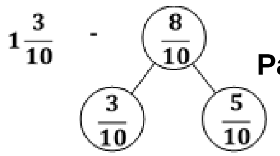
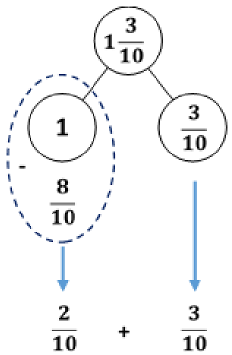
Number and Place Value	Addition and Subtraction	Multiplication and Division
	Then adapted to decimal and fractional part whole as well as measures such as time and money. For example, $1\frac{2}{7} - \frac{5}{7} =$  I can regroup the subtrahend $\frac{5}{7}$ into $\frac{2}{7}$ and $\frac{3}{7}$. Then I can take away the $\frac{2}{7}$ leaving $\frac{7}{7}$ or 1 and finally take away $\frac{3}{7}$.  ... or I could regroup the minuend, subtract from the 1 and then recombine.	

Number and Place Value	Addition and Subtraction	Multiplication and Division
	Extend into UKS2 by converting fractions into equivalents with common denominators. Beginning with conversions where no regrouping is required. For example, $\frac{2}{10} - \frac{1}{20} =$ Progress to examples where regrouping would be a valid strategy. For example, $1\frac{3}{10} - \frac{4}{5} =$ Pupils will have to know that $\frac{4}{5} = \frac{8}{10}$ before they can solve the calculation. Then they could regroup either the subtrahend or the minuend.  For example, $1\frac{3}{10} - \frac{8}{10} =$  **Partitioning the subtrahend** **Partitioning the minuend** 	

Number and Place Value	Addition and Subtraction	Multiplication and Division
	Core concept: CONSERVATION **Reordering and finding complements** across a range of numbers. For example: 47 + 603 0.45 + 1.63 0.15 + 1.85 £3.99 + £7.80 + £2.01 Two decimal numbers add together to make a total of 1. One number is 0.0006. What is the other number?	**Core concept: CONSERVATION** **Core skill: REARRANGING** **Factorisation** drawing on the associative law for multiplication and related division facts. For example, 24 x 3 = 12 x 3 x 2  Two and twelve are factors of 24 and I find it easier to calculate 12 x 3 first and then double it. **Doubling and halving** 12 x 2.5 = 12 x 2.5 = 6 x 5. I halved the 12 and doubled the 2.5 to make the calculation easier. 16 x 6 ¼ = 16 x 6 ¼ = 8 x 12 ½ = 4 x 25 = 100 I can make this easier for me by doubling and doubling again the 6 ¼. This means I have to halve and halve again the 16 to maintain the area. Now I get 4 x 25 = 100. *Application to KS2 example (Q11 paper 1 2016):* *71 x 8 = 142 x 4* *= 284 x 2*

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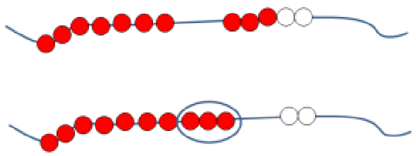
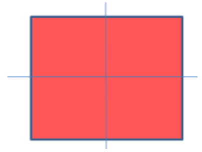
Number and Place Value	Addition and Subtraction	Multiplication and Division
	Core concept: CONSERVATION and COMPARISON **Core skill: REBALANCING** **Equal sum** drawing out the concept of equality when rebalancing the numbers in an addition calculation.  Pupils use bead strings to demonstrate that: 7 + 5 = 10 + 2 Apply concept to range of numbers and missing number problems. For example, 24 + ☐ = 30 + 3. **<u>See Year 3 and 4 examples</u>** These should include rehearsal using calculations such as: 39 + 52 345 + 198 0.39 + 6.54 5.1 + 2.7 = ☐ + 4.8	**Halving and halving for division** Once pupils are confident with the ‘halve and double’ strategy for multiplication, they will try to apply it to division and will need to understand why their answers do not make sense. Stress again the importance of estimation. Investigate the principle of halving and halving with pupils.  72 ÷ 4 = (72 ÷ 2) ÷ 2 When I am dividing by 4, I like to halve the number and halve it again. This strategy is best explored through practical contexts so pupils can clearly see that even though the dividend and the divisor are changing the quotient remains constant.

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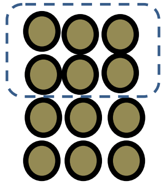
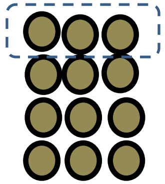
Number and Place Value	Addition and Subtraction	Multiplication and Division
	Ensure pupils are secure with the concept of **equal sum** before considering questions such as: 7834 + 79,996 79,996 is 4 away from 80,000. I can rebalance the sum by taking 4 from 7834 and giving it to the 79,996. Now I have 80,000 + 7,830 = 87,830. **Compensation** with the same calculation supports pupil's multi-strategy approach. Pupils can continue to ***evaluate*** strategies. 7834 + 79,996 Adding 79,996 is like adding 80,000 and subtracting 4. I can do 80,000 + 7834 - 4 = 87,830 Improve multi-strategy approaches by asking for two different ways of solving calculations such as: ☐ = 5,756 + 8,643 16.98 + 23.214 = ☐	For example, If I shared 12 cookies among 4 children each child would get 3 cookies. 12 ÷ 4 = 3  I can also see that 6 cookies shared between 2 people would give the same group size. The size of the group hasn't changed. So 12 ÷ 4 can be changed into 6 ÷ 2.  As I am trying to find out the group size, I can also see that 3 ÷ 1 gives me the group size. So 12 ÷ 4 can be thought of as 6 ÷ 3 and 3 ÷ 1. I can see all of these in the array. Applying this conceptual understanding to larger numbers encourages playfulness with division. *364 ÷ 16 =* 182 ÷ 8 = 91 ÷ 4 = 45.5 ÷ 2 = 22.75 I saw that I could halve both the dividend and the divisor, so I did to see if it made it easier. Then I realised that I could halve them again and again. *Apply core concepts in the context of fractions.*

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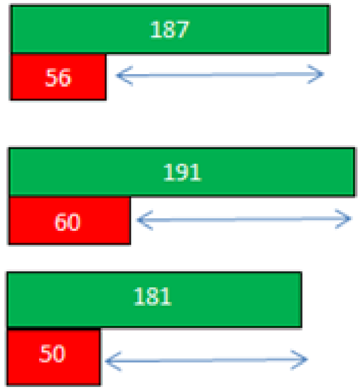
Number and Place Value	Addition and Subtraction	Multiplication and Division

Addition and Subtraction

Equal difference using comparison drawing out the concept that **adding** or **subtracting** the same quantity from both the **subtrahend** and **minuend** will maintain the difference between the numbers.



I can add 4 or take 6 away from each of the numbers and the difference will be the same.

Remember to rehearse simple calculations such as 367 - 9 before applying to a range of numbers.

132,457 - 11,999 =

11,999 is nearly 12,000. If I add one to each number the difference will stay equal. Now my calculation is 132,458 - 12,000 =

Multiplication and Division

Core concept: UNITISING

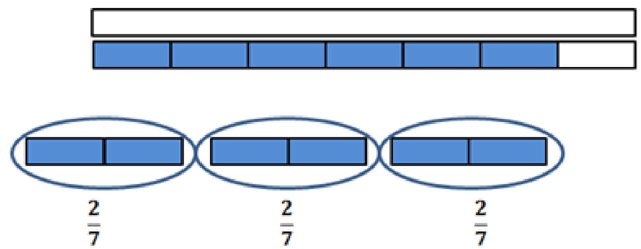
Core skill: REGROUPING

Division of fractions by integers drawing out the concept of multiple groups of the numerator before teaching a rule. Ensure that pupils always refer to the whole.

I know that 12 ÷ 3 can be thought of as 'If I share 12 equally between 3 groups, how many in each group?'

So $\frac{6}{7}$ ÷ 3 can be thought of as, 'If I share $\frac{6}{7}$ equally between 3 groups, how many in each group?'

$\frac{6}{7} \div 3$



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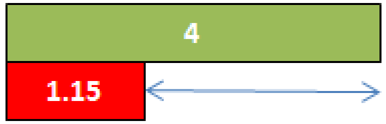
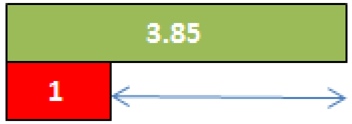
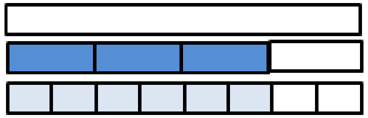
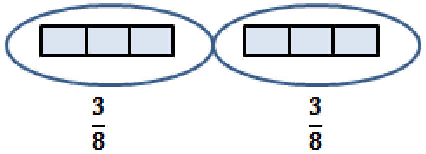
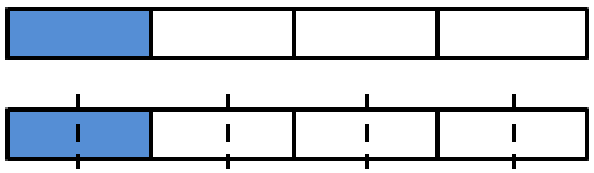
Number and Place Value	Addition and Subtraction	Multiplication and Division
	Use a range of examples. ☐ = 4 - 1.15   It is easier if I subtract 0.15 from each number. The difference will stay the same. Now my calculation is 3.85 - 1 = **<u>Compare</u>** this to **compensation** 132,457 - 11,999 = Subtracting 11,999 is like subtracting 12,000 and then adding 1. Now my calculation is 132,457 - 12,000 + 1 =	Progress to dividing fractions in which the fraction needs converting. $\frac{3}{4} \div 2$   ¾ ÷ 2 can be understood as: “If I share ¾ equally between 2 groups, how many in each group?”

Number and Place Value	Addition and Subtraction	Multiplication and Division
		Multiplication of fractions by fractions **Equal groups** I know that 3 x 4 could mean 3 groups of 4. So ½ x ¼ means half a group of ¼. ½ x ¼ =  When we find half of any number, we divide it by two. The blue part has a value of $\frac{1}{4}$. When I halve it, it makes $\frac{1}{8}$. Pupils should focus upon the denominators and reason why, when multiplied, we find the product of the denominators. Once understood pupils can employ the rule.

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Number and Place Value	Addition and Subtraction	Multiplication and Division
		Halve and double The '**halve and double'** rule can be applied to fractions. Pupils have already secured conceptual understanding of this rule, for example: 5 x 4 = 10 x 2 = 20 x 1 Apply this understanding to fractions, for example: ½ x ¼ = If we double the first term and halve the second, we can transform the calculation to: $1 \times \frac{1}{8} = \frac{1}{8}$ *For further detail regarding the multiplication and division of fractions refer to the 'HfL Bar Modelling Progression' document.*

Upper KS2 examples

Place Value

937 + 100 1969 + 100 546 - 40
1.7 + 0.05 40 000 - 500
246 ÷ 1 100 x 217 0.4 ÷ 10
1.68 x 100 100 x 100

Examples from 2016 KS2 and Sample Papers

435 - 30 979 + 100 3.005 + 6.12 2.15 + 0.05
100 x 412 0.9 ÷ 10 1.28 x 100 50,000 – 500
10 x 100

Two decimal numbers add together to equal 1 One of the numbers is 0.007. What is the other number?

Circle two numbers that added together make 0.25
0.05 0.23 0.2 0.5

Circle two numbers that multiply together to equal 1 million 200 2,000 5,000 50,000

Write the number that is 5 less than 10 million

Write the number that is one hundred thousand less than six million

Round 124,531 to the nearest 10,000, 1,000, 100

Think Regroup

58 + 6 5 + 47 630 + 73 680 + 78
560 + 89 8900 + 230
74 - 7 97 - 8 320 - 50 2300 - 600
3400 - 1700

5 - 2.65 8.1 - 2.75 $1\frac{2}{5}+\frac{3}{10}=$ $1\frac{3}{10}-\frac{2}{5}=$
£3367.40 - £1021.23

Examples from 2016 KS2 and Sample Papers

4 - 1.15 $1\frac{4}{5}+\frac{3}{10}$ $1\frac{1}{4}+\frac{1}{3}$ $1\frac{1}{5}-\frac{1}{4}$ $\frac{3}{4}+\frac{7}{8}=$

5,756 + 8,643 936 + 285

Compensation

56 + 8 72 + 9 56 - 8 72 - 9
371 + 18 255 + 49 304 + 299
673 - 99 854 - 398 3720 - 996
0.71 + 0.09 0.56 + 0.08 0.34 - 0.09
£1.17 + £0.39 £8.89 - £4.99

Examples from 2016 KS2 and Sample Papers

468 - 9 472 - 9 15.98 + 26.314
12 - 6.01 15.4 - 8.88

Rebalancing - Equal sum

56 + 8 72 + 9 371 + 18 255 + 49
304 + 267
£37.67 + £3.85 563 + 397 890,488 + 4,890
229,899 + 31,321

Examples from 2016 KS2 and Sample Papers

89,994 + 7,643 936 + 285 89,994 + 7,643

Rebalancing - Equal difference

85 - 18 42 - 17 88 - 43 437 - 103
819 - 504 532,525 - 9897
£122.56 - £87.99 9.1 - 6.7 15.3 - 5.7

Examples from 2016 KS2 and Sample Papers

468 - 9 472 - 9 122,456 - 11,999
4 - 1.15 12 - 6.01
15.4 - 8.88 234,897 - 45,996

Think Partition for x and ÷

32 x 4 29 x 2 122 x 4 4.6 x 2
75 x 3 8.3 x 6 39 x 7
3.3 x 7 5 x 49 4 x 198 96 x 0.3

Examples from 2016 KS2 and Sample Papers

15 x 6.1 24 x 3 1.52 x 6 7,505 ÷ 5
17 x 1½

Make links to doubling and halving

50 x 28 86 x 50 500 x 70 18 x 2.5
86 x 2.5 160 x 35 500 x 88 1.5 x 6.6
0.5 x 120 4.5 x 2.2 15% x 346 75% x 220

Examples from 2016 KS2 and Sample Papers

15% x 440 $\frac{2}{5}$ x 140 24 x 3
20% of 1500 95% of 240

Multiplying and dividing fractions

Examples from 2016 KS2 and Sample Papers

$\frac{3}{5}\div 3$ $\frac{2}{5}\div 2$ $\frac{3}{4}\div 2$ $\frac{2}{5}$ x 140 $\frac{1}{4}\times\frac{1}{8}$

Upper KS2 examples *(continued)*

Re-ordering and finding complements	**x and ÷ by powers of 10**	
11 + 59 33 + 57 14 + 90 + 86 290 + 310 1.15 + 2.55 0.8 + 0.26 *Examples from 2016 KS2 and Sample Papers* 1,034 + 586 2.15 + 0.05 Circle two numbers that added together make 0.25 0.05 0.23 0.2 0.5	10 x 53 87 x 10 1000 x 14 100 x 8.3 100 x 0.41 30 x 3 7 x 0.3 30 x 30 30 x 70 567 ÷ 100 36 ÷ 10 0.5 ÷ 10 280 ÷ 4 5600 ÷ 80 30 = □ ÷ 12 270 ÷ 9 = □ ÷ 0.9 7 x 0.001 1.8 ÷ 0.1 3.25 ÷ 0.00001 Circle two numbers that multiply together to equal 10 million 200 2,000 5,000 50,000 *Examples from 2016 KS2 and Sample Papers* 1440 ÷ 12 630 ÷ 9 1,320 ÷ 12 0.9 ÷ 10 20% of 1,800 20% of 1500 7,505 ÷ 5 95% of 240 100 x 412 0.9 ÷ 10 1.28 x 100 50,000 - 500 10 x 100 Circle two numbers that multiply together to equal 1 million 200 2,000 5,000 50,000	

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